

AB Calculus: Polynomial Functions and Inequality Solutions | Rational Function Inequalities

Learning Target: I can graph polynomials with roots of multiplicity, recognize local & global behavior, and solve inequalities. I can solve rational inequalities by using the graph of the related polynomial.

- A) Graph the polynomial function on the axes provided where $A > 0$. B) State the end-behavior by completing the limit statements. C) Use the graph to state the intervals for x on which the polynomial is greater than or equal to zero using correct mathematical notation. D) Solve the rational inequality where $A > 0$ & $B > 0$.

Polynomial Inequality	Graph of Polynomial Function	Rational Inequality
1) $f(x) = A(x+4)(x-3)$ $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$ Solve $f(x) \geq 0$ $x \in \underline{\hspace{4cm}}$		$f(x) = \frac{A(x+4)}{B(x-3)} \geq 0$ $x \in \underline{\hspace{4cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$
2) $f(x) = A(x+4)(x+1)(x-3)$ $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$ Solve $f(x) \geq 0$ $x \in \underline{\hspace{4cm}}$		$f(x) = \frac{A(x+4)}{B(x+1)(x-3)} \geq 0$ $x \in \underline{\hspace{4cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$
3) $f(x) = (x+4)^2(x+1)(x-3)$ $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$ Solve $f(x) \geq 0$ $x \in \underline{\hspace{4cm}}$		$f(x) = \frac{A(x-3)}{B(x+4)^2(x+1)} \geq 0$ $x \in \underline{\hspace{4cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$
4) $f(x) = (x+4)(x+1)^3(x-3)$ $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$ Solve $f(x) \geq 0$ $x \in \underline{\hspace{4cm}}$		$f(x) = \frac{A(x+4)(x+1)^3}{B(x-3)} \geq 0$ $x \in \underline{\hspace{4cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$
5) $f(x) = (x+4)^2(x+1)^2(x-3)$ $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$ Solve $f(x) \geq 0$ $x \in \underline{\hspace{4cm}}$		$f(x) = \frac{A(x+1)^2}{B(x+4)^2(x-3)} \geq 0$ $x \in \underline{\hspace{4cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$
6) $f(x) = (x+4)^3(x+1)(x-3)^2$ $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$ Solve $f(x) \geq 0$ $x \in \underline{\hspace{4cm}}$		$f(x) = \frac{A(x-3)^2(x+1)}{B(x+4)^3} \geq 0$ $x \in \underline{\hspace{4cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$
7) $f(x) = (x+4)^3(x+1)^2(x-3)$ $\lim_{x \rightarrow -\infty} f(x) = \underline{\hspace{2cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$ Solve $f(x) \geq 0$ $x \in \underline{\hspace{4cm}}$		$f(x) = \frac{A(x+4)^3}{B(x+1)^2(x-3)} \geq 0$ $x \in \underline{\hspace{4cm}}$ $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$

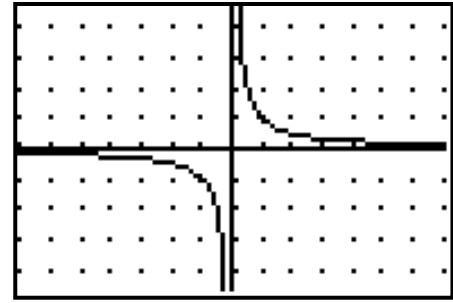
AB Calculus: Rational Functions and Basic Transformations

Learning Target: I know how to

- 1) Identify characteristics of a rational function from graph and equation.

Characteristics include: vertical asymptotes, horizontal or oblique asymptotes, holes, x-intercept(s) and y-intercept.

- 2) Write an equation for a rational function by identifying the horizontal asymptote, vertical asymptotes and the behavior of the graph near each VA.



Here is the toolkit function for the rational function $f(x) = \frac{1}{x}$. Transformations of this function in terms of the equation

$g(x) = Af[B(x-C)] + D$ are exactly the same as transformations you have done for all toolkit functions like

$f(x) = x^2$, $f(x) = |x|$, $f(x) = \sqrt{x}$ and $f(x) = x^3$. Generally speaking, Mrs. Eagen will only be concerned that you

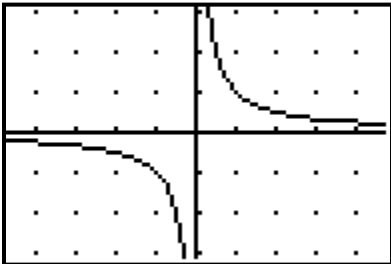
know $g(x) = Af(x-C) + D$ for the rational function transformations.

When you draw your graphs you will include the vertical and horizontal asymptotes as dashed lines, oblique or slant asymptotes as dashed lines, and holes as open circles.

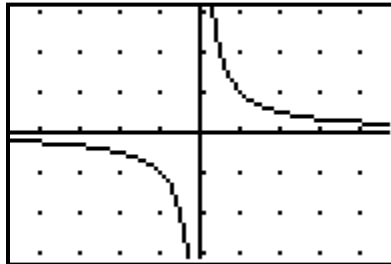
Basic Transformations of the Rational Function: Given the graph of the rational function: $f(x) = \frac{1}{x}$, sketch on the

same grid the basic transformation given by the equation.

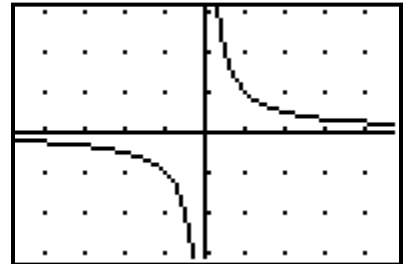
1) $g(x) = -f(x) = \frac{-1}{x}$



2) $g(x) = [f(x)]^2 = \frac{1}{x^2}$

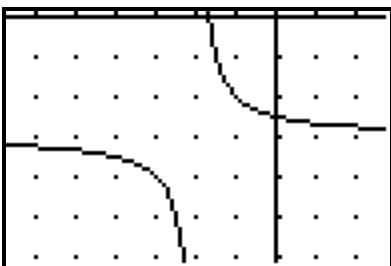


3) $g(x) = -[f(x)]^2 = \frac{-1}{x^2}$

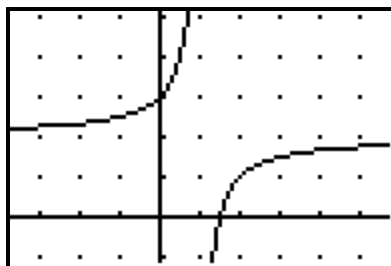


Sketch in the vertical and horizontal asymptotes then write these equations for the rational functions. Let A=1.

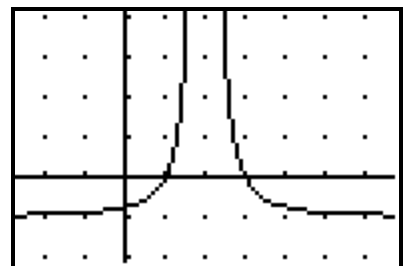
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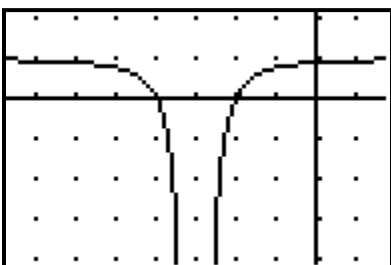
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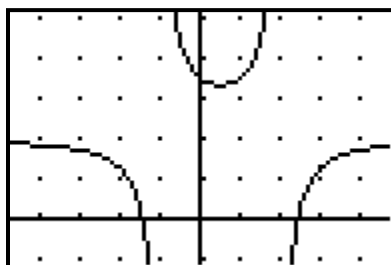
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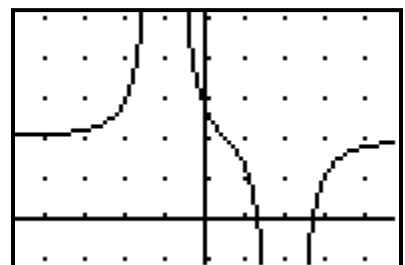
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8)



9)



AB Calculus: Rational Functions and Equivalent Forms

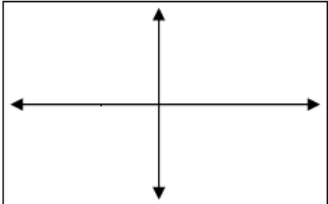
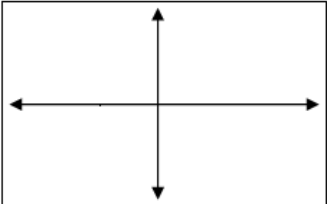
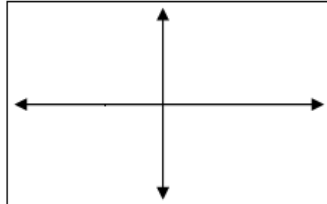
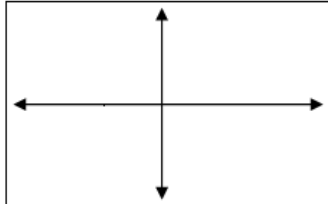
Learning Target: I can

1) Re-express a rational function using algebraic techniques: factoring & division.

$$y = \frac{\text{numerator}}{\text{denominator}} \quad y = \frac{\text{product of factors}}{\text{product of factors}} \quad y = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

2) Identify when a hole occurs and the (x, y) coordinate of hole from factored form.

3) Recognize the Quotient as the “end behavior asymptote,” either the horizontal or oblique asymptote.

10) $f(x) = \frac{2x-5}{x-3}$	11) $f(x) = \frac{(2x-5)(x+1)}{(x-3)(x+1)}$	12) $f(x) = \frac{x^2-2x-2}{x-3}$	13) $f(x) = \frac{2x^2-3x-2}{x^2-x-2}$
Rewrite:			Factor: Rewrite:
Domain:			
VA:			
HA/OA:			
Range:			
Evaluate: $f(-1) =$	Hole:	Hole:	Hole:
Y-intercept:			
X-intercepts:			
Sketch:			
			

AB Calculus: Rational Functions and End Behavior

Learning Target: Given a rational function, $y = \frac{a_1x^m + a_2x^{m-1} + \dots + a_{m-1}x + a_m}{b_1x^n + b_2x^{n-1} + \dots + b_{n-1}x + b_n}$ I can use the "end-behavior-model" to

determine the long-run end-behavior of the graph and decide if the rational function has:

- a horizontal asymptote at $y = 0$
- a horizontal asymptote at $y = c$ where c is a constant.
- an oblique asymptote modeled by the "end-behavior model" given by: $y = \frac{a_1x^m}{b_1x^n}$.

Examine the following rational functions and complete part a, b, & c for each.

- Write an **end-behavior model** for each rational function. Simplify by reducing common factors when possible.
- Complete the behavior phrases: "as $x \rightarrow -\infty$, $y \rightarrow \underline{\hspace{1cm}}$ " or "as $x \rightarrow \infty$, $y \rightarrow \underline{\hspace{1cm}}$ " for each **EBM**.
- Complete long division, and re-express the rational function as: $y = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$.
- Compare the end behavior model from part (a) and the quotient from part (b).

14) $f(x) = \frac{6x^2 + 4x + 19}{2x^2 + x + 5}$

15) $f(x) = \frac{8x^3 + 5x - 1}{2x^4 - 3x^3 - 2}$

16) $f(x) = \frac{9x^4 - 6x^3 - 1}{3x^2 + 2}$

Summarize your conclusions from the 3 examples above.

Given a rational function $y = \frac{a_1x^m + a_2x^{m-1} + \dots + a_{m-1}x + a_m}{b_1x^n + b_2x^{n-1} + \dots + b_{n-1}x + b_n}$ the **end-behavior model** is _____.

- The graph will have a **horizontal asymptote** at $y = 0$ when _____.
- The graph will have a **horizontal asymptote** at _____ when _____.
- The graph will have an **oblique asymptote** of _____ when _____.
An **oblique asymptote** is also called a **slant asymptote**.

Use your short-cuts from the summary to state the following for each rational function.

- end-behavior model
- complete the behavior phrases: "as $x \rightarrow -\infty$, $y \rightarrow \underline{\hspace{1cm}}$ " or "as $x \rightarrow \infty$, $y \rightarrow \underline{\hspace{1cm}}$ "
- horizontal asymptote, if applicable

17) $f(x) = \frac{3x^7 + 5x - 1}{6x^7 - 3x^2 + 5}$

18) $f(x) = \frac{3x^7 + 5x - 1}{6x^8 - 3x^2 + 5}$

19) $f(x) = \frac{3x^8 + 5x - 1}{6x^7 - 3x^2 + 5}$