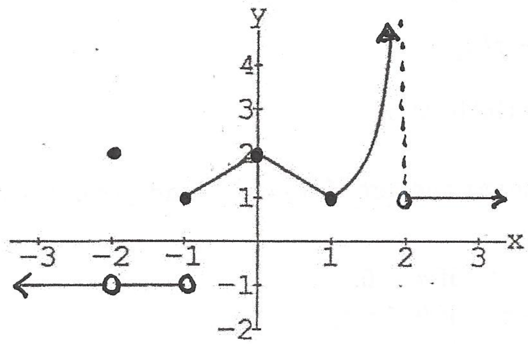
DAY 36: HW #1-9, 11

§2.6 WS: Continuity & Differentiability

Show work on a separate sheet of paper in your notebook

1. The graph of a function f is given below.



a. For what numbers x in $[-3, 3]$ is f *not* continuous at x? Explain your answer.

b. For what numbers x in $[-3, 3]$ is f *not* differentiable at x? Explain your answer.

2. For some given non-zero number a, define

$$f(x) = \begin{cases} (x^2 - a^2)/(x - a) & \text{if } x \neq a, \\ 0 & \text{if } x = a. \end{cases}$$

a. Is f defined at a?

b. Does $\lim_{x \to a} f(x)$ exist? Justify your answer.

c. Is f continuous at a? Justify your answer.

d. Is f differentiable at a? Justify your answer.

3. For $x \neq 0$ define $f(x) = \dfrac{\sin x}{x}$. Is it possible to define $f(0)$ so that f is continuous at 0? Explain your answer.

4. If $\lim_{x \to a} f(x) = L$, which of the following statements, if any, MUST be true? Justify your answer.

(i) f is defined at a.

(ii) $f(a) = L$.

(iii) f is continuous at a.

(iv) f is differentiable at a.

5. If a function f is continuous at a, which of the following statements, if any, MUST be true? Justify your answer.

   (i) f is defined at a.

   (ii) $\lim_{x\to a} f(x)$ exists.

   (iii) $\lim_{x\to a} f(x) = f(a)$

   (iv) f is differentiable at a.

6. Suppose f is continuous and $\lim_{x\to a} f(x) = L$. Find $f(a)$. Justify your answer.

7. Let $f(x) = \begin{cases} 2 & \text{if } x < 0, \\ 3 - x & \text{if } 0 \le x \le 1, \\ x^2 + 1 & \text{if } x > 1. \end{cases}$

   a. Is f continuous at $x = 0$? Justify your answer.

   b. Is f continuous at $x = 1$? Justify your answer.

8. Determine whether the following statements MUST be true or are at least SOMETIMES false. Justify your answers.

   a. If f is continuous at a point x, then it is differentiable at x.

   b. If f is differentiable at a point x, then it is continuous at x.

9. Let $f(x) = \begin{cases} ax & \text{if } x \le 1, \\ bx^2 + x + 1 & \text{if } x > 1. \end{cases}$

   a. Find all choices of a and b such that f is continuous at $x = 1$.

   b. Draw the graph of f when $a = 1$ and $b = -1$.

   c. Find values of a and b such that f is differentiable at $x = 1$.

   d. Draw the graph of f for the values of a and b found in part c.

10. George takes a trip from St. Louis to Chicago. He leaves at 9 AM on Monday and arrives at 2 PM that day. He returns on Tuesday, leaving at 9 AM and arriving back in St. Louis at 2 PM, retracing exactly the same route. Show that there is a point on the road through which he passes at the same time both days.

11. Determine whether the following statements are always true or are at least sometimes false. Justify your answers.

   a. If $f(1) < 0$ and $f(2) > 0$, then there must be a point z in $(1, 2)$ such that $f(z) = 0$.

   b. If f is continuous on $[1, 2]$, $f(1) < 0$ and $f(2) > 0$, then there must be a point z in $(1, 2)$ such that $f(z) = 0$.

   c. If f is continuous on $[1, 2]$ and there is a point z in $(1, 2)$ such that $f(z) = 0$, then $f(1)$ and $f(2)$ must have different signs.

   d. If f has no zeros and is continuous on $[1, 2]$, then $f(1)$ and $f(2)$ have the same sign.