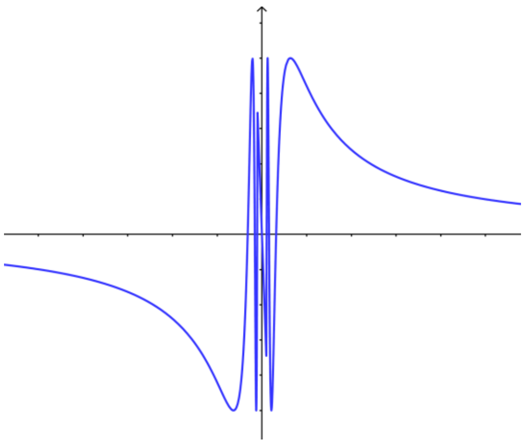
Oscillating Discontinuity

Consider the graph of $f(x) = \sin\left(\frac{1}{x}\right)$.



Complete this table:

x	$f(x) = \sin\left(\frac{1}{x}\right)$
-0.1	
-0.01	
-0.001	
-0.0001	
0.0001	
0.001	
0.01	
0.1	

Explain what is happening as the *x*-values get closer and closer to 0.

What does this tell us about $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$?

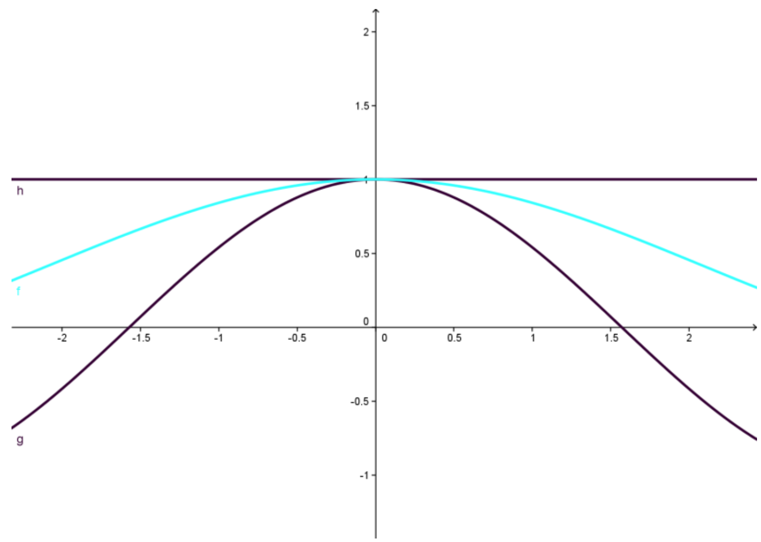
The Sandwich Theorem

If $g(x) \le f(x) \le h(x)$ **for all** $x \ne c$ **in some interval about *c*,**

and

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L,$$

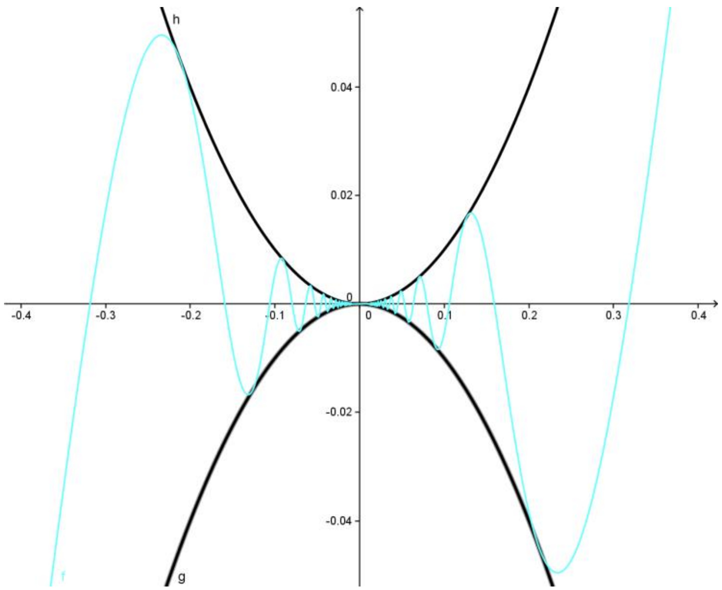
then $\lim_{x \to c} f(x) = L.$



Graph $f(x) = x^2 \sin(\frac{1}{x})$.

What two functions "bound" *f*(*x*)?

Why do these functions "bound" *f*(*x*)?



Sandwich Theorem Worksheet

1. Prove that $\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0.$

2. Prove that $\lim_{x \to 0} x^2 \sin\left(\frac{5}{x}\right) = 0.$

3. Prove that $\lim_{x \to 0} x^2 \sin\left(\frac{50\pi}{\sqrt[3]{x}}\right) = 0.$

4. Sketch the graphs of
   $y = 1 - x^2$, $y = \cos x$, and $y = f(x)$
   where *f* is a function that satisfies the inequalities $1 - x^2 \le f(x) \le \cos x$
   for all x in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.
   What can you say about the limit of $f(x)$ as x approaches 0? Explain your reasoning.

5. If $3x \le f(x) \le x^3 + 2$ for $0 \le x \le 2$, evaluate $\lim_{x \to 1} f(x)$.