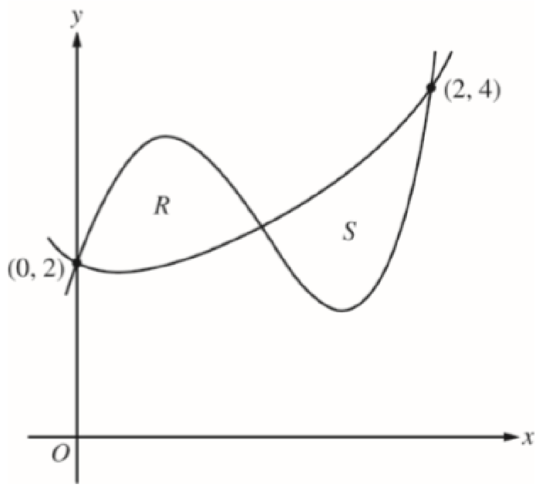
FRQ: Area and Volume Tuesday 3 April 2018 CLASSWORK: 4 FRQs Partner Collaborative

Show all work on a neatly separate sheet of paper. Your classmates will be grading your work using the AP scoring rubric

2015 #2 Calculator Active



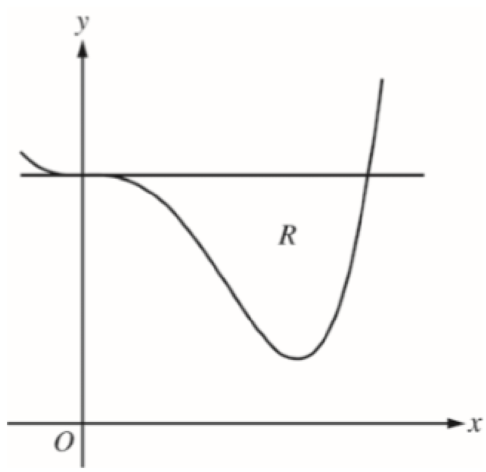
2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2 - 2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

   (a) Find the sum of the areas of regions R and S.

   (b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

   (c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

2014 #2 Calculator Active



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

   (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

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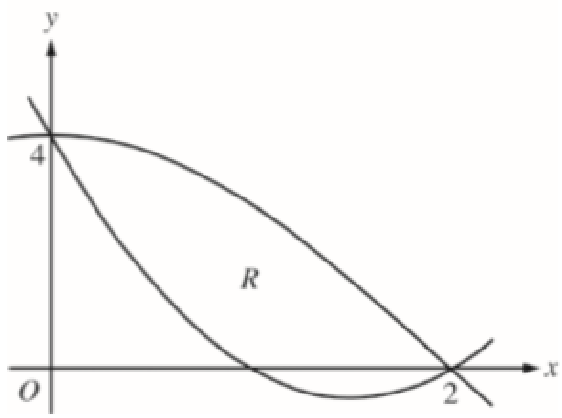
2013 #5 NON-Calculator



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

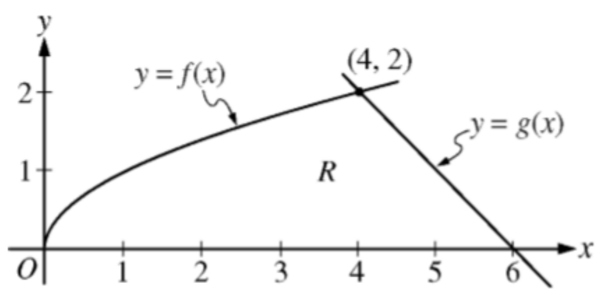
(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

2011 #3 NON-Calculator

No calculator is allowed for these problems.



3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

Complete work for all questions on a separate sheet of paper.

1. (calculator not allowed)
The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

2. (calculator not allowed)
What is the area of the region in the first quadrant bounded by the graph of $y = e^{\frac{x}{2}}$ and the line $x = 2$?

(A) $2e-2$ (B) $2e$ (C) $\frac{e}{2}-1$ (D) $\frac{e-1}{2}$ (E) $e-1$

3. (calculator allowed)
What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

4. (calculator not allowed)
The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

5. (calculator not allowed)
Let R be the region in the first quadrant bounded above by the graph of $y = \sqrt{x}$ and below by the graph of $y = x^2$. R is the base of a solid whose cross sections perpendicular to the $y-$axis are squares. Which of the following gives the volume of the solid?

(A) $\int_0^1 \left(\sqrt{x} - x^2\right)^2 dx$ (B) $\int_0^1 \left(x - x^4\right) dx$ (C) $\int_0^1 \left(\sqrt{y} - y^2\right)^2 dy$ (D) $\int_0^1 \left(\sqrt{y} - y^2\right) dy$

6. (calculator allowed)

Let R be the region in the first and second quadrants bounded above by the graph of $y=\dfrac{20}{1+x^2}$ and below by the horizontal line $y=2$. R is the base of a solid whose cross sections perpendicular to the $x-$axis are semicircles. What is the volume of the solid?

(A) 29.815 (B) 174.268 (C) 348.537 (D) 443.771

7. (calculator not allowed)

The functions f and g are given by $f(x)=2\sqrt{x}$ and $g(x)=x-3$. Let R be the region bounded by the $x-$axis and the graphs of f and g. The graphs of f and g intersect in the first quadrant at the point $(9,6)$. Which of the following gives the volume of the solid generated when R is revolved about the $x-$axis?

(A) $4\pi\int_0^3 x\,dx+\pi\int_3^9\left(4x-(x-3)^2\right)dx$

(C) $\pi\int_0^9\left(2\sqrt{x}-(x-3)\right)^2 dx$

(B) $4\pi\int_0^3 x\,dx+\pi\int_3^9\left(2\sqrt{x}-(x-3)\right)^2 dx$

(D) $\pi\int_0^9\left(4x-(x-3)^2\right)dx$

8. (calculator not allowed)

The function f is given by $f(x)=\ln x$. Which of the following limits is equal to the area between the graph of $f(x)$ and the $x-$axis from $x=1$ to $x=3$?

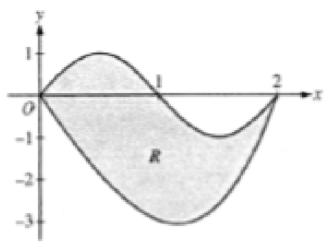
(A) $\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(1+\frac{2k}{n}\right)\frac{2}{n}$

(C) $\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(1+\frac{2k}{n}\right)\frac{1}{n}$

(B) $\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(1+\frac{2k}{n}\cdot\frac{2}{n}\right)$

(D) $\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(\frac{2k}{n}\right)\frac{2}{n}$

13. (calculator allowed)



Let R be the region bounded by the graphs of $y=\sin(\pi x)$ and $y=x^3-4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y=-2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x)=3-x$. Find the volume of water in the pond.

Complete work for all questions on a separate sheet of paper.

Calculator Skills for the AP Exam

Using a graphing calculator on the AP Calculus exam
Students are expected to know how to use their graphing calculators on the AP Calculus exams proficiently to accomplish the following four capabilities:

- Plot the graph of a function within an arbitrary viewing window
- Find the zeros of functions (solve equations numerically)
- Numerically calculate the derivative of a function
- Numerically calculate the value of a definite integral

A student's ability to use a graphing calculator effectively will be tested on the calculator active portions of the AP Exam. Students should know when and how to properly use their graphing calculators on the calculator active questions.

- **One very common mistake that hurts a student's score is trying to work out a complex problem by hand when a calculator is available!**
- **Students need to avoid preliminary rounding and use the storing feature of their calculator to aid in the accuracy of answers.**
- **When possible, refer to the names of functions given in the problem (such as *f(x)* or *g(x)*) to avoid making careless errors when setting up solutions.**

IMPORTANT NOTE: Graphing calculators are a valuable tool for numeric calculations and to understand the behavior of a graph but CANNOT be used as justification on free response questions. An appropriate mathematical justification and/or explanation is necessary. Also, when using a graphing calculator on the free response questions, avoid writing down calculator syntax on the exam. Credit will not be awarded for simply writing down what is "typed" into the calculator. Proper calculus notation must be used.

Calculator skills that are required or beneficial for the AP exam:

- Determine the point of intersection
- Determine the zero of a function
- Store and recall values
- Determine points of extrema
- Calculate numerical derivatives
- Calculate definite integrals
- Graph derivative(s) given a function

Complete work for all questions on a separate sheet of paper.

Multiple Choice

1. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t=0$ to the time $t=8$?

   (A) $\frac{a(8)-a(0)}{8}$ (B) $\frac{1}{8}\int_0^8 v(t)\,dt$ (C) $\frac{1}{2}\int_0^8 |v(t)|\,dt$ (D) $\frac{1}{2}\int_0^8 v(t)\,dt$ (E) $\frac{v(0)-v(8)}{2}$

2. Let $f'(x)=\frac{\ln x}{e^x}-\cos x$ for $1<x<6$. On what intervals is f concave down?

   (A) (1.481, 4.726) (B) (1, 3.105) (C) (3.105, 6) (D) (1, 4.726) (E) (1, 1.481),(4.726, 6)

3. Water is pumped into a tank at a rate of $r(t)=30\left(1-e^{-0.16t}\right)$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

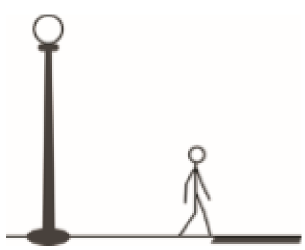
   (A) 380 gallons (B) 420 gallons (C) 829 gallons (D) 1220 gallons (E) 1376 gallons

4. Two particles start at the origin and move along the x-axis. For $0\le t\le 4$, their respective position functions are given to be $x_1=\ln\left(t^2\right)$ and $x_2=(t-5)^2$. For what value of t does the acceleration of x_1 equal the velocity of x_2?

   (A) 1.060 (B) 0.470 (C) 3.960 (D) 2.039 (E) None

5. The average value of the function $f(x)=\cos(x^2)$ on the closed interval $[0, 2]$ is

   (A) 0.231 (B) 0.461 (C) 0.780 (D) 0.977 (E) 1.253



6. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

   (A) 1.5 ft/sec (B) 2.667 ft/sec (C) 3.75 ft/sec (D) 6 ft/sec (E) 10 ft/sec

7. If c satisfies the conclusion of the Mean Value Theorem for $f(x)=\sin^{-1}x$ on the interval $0\le x\le 1$, then c is

   (A) 0.500 (B) 0.771 (C) 0.785 (D) 1 (E) 1.186

8. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \dfrac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 (E) 11.710

9. Given $f(x) = \int_1^x \dfrac{t^2}{(e^t+1)(\sin t)}\,dt$; $f''(1) =$

(A) 0 (B) 0.200 (C) 0.320 (D) 0.341 (E) does not exist

10. The area of the region bounded by the curves $y = e^{-x}$, $y = \ln x$, and the line $x = 1$ is

(A) 0.042 (B) 0.054 (C) 0.096 (D) 0.728 (E) 1.686

11. Let $g(x)$ be the function given by $g(x) = \int_0^x e^t(t^2-1)\,dt$. Which of the following must be true?

I. g is decreasing on (0,1)
II. g is decreasing on (1,2)
III. $g(2) > 0$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

12. Let R be the region in the first quadrant enclosed by the graphs $y = x^4 + 1$ and $y = x + 16$. The volume of the solid generated by revolving R about the x-axis is

(A) 25.616 (B) 80.475 (C) 507.539 (D) 1594.480 (E) 3188.959

Complete work for all questions on a separate sheet of paper.

13.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

15. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure.

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$.

(d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

Complete work for all questions on a separate sheet of paper.

Fundamental Theorem of Calculus

Students should be able to:

- Use the fundamental theorem to evaluate definite integrals.
- Use various forms of the fundamental theorem in application situations.
- Calculate the average value of a function over a particular interval.
- Use the other fundamental theorem.

Multiple Choice

1. (calculator not allowed)

$\int_{-2}^{x}\left(4t^3-2t\right)dt=$

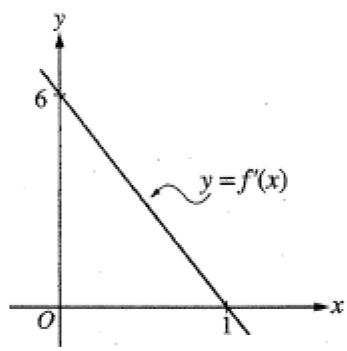
(A) x^4-x^2 (B) x^4-x^2-12 (C) $4x^3-2x$ (D) $4x^3-2x-28$

2. (calculator not allowed)

What is the average value of y for the part of the curve $y=3x-x^2$ which is in the first quadrant?

(A) -6 (B) -2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

3. (calculator not allowed)



The graph of f', the derivative of f, is the line shown in the figure above. If $f(0)=5$, then $f(1)=$

(A) 0 (B) 3 (C) 6 (D) 8 (E) 11

4. (calculator not allowed)

$\frac{d}{dx}\int_0^{x^2}\sin(t^3)dt=$

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x\sin(x^3)$ (E) $2x\sin(x^6)$

5. (calculator not allowed)

Let $f(x)=\int_{-2}^{x^2-3x}e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

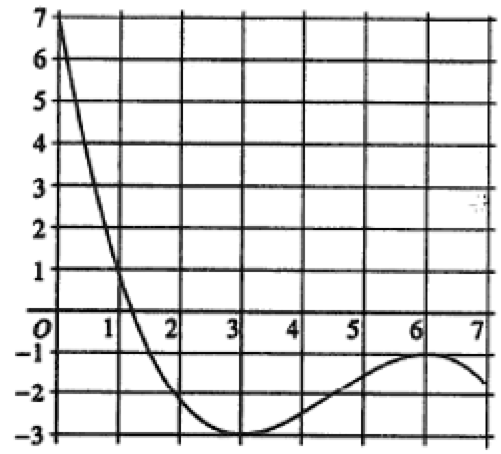
(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

6. (calculator not allowed)

$\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

(A) $-2(\sqrt{2}-1)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$ (D) $2(\sqrt{2}-1)$ (E) $2(\sqrt{2}+1)$

7. (calculator not allowed)



Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0 (B) −1 (C) −2 (D) −3 (E) −6

8. (calculator allowed)

Let h be the function defined by $h(x) = \frac{1}{\sqrt{x^5+1}}$. If g is an antiderivative of h and $g(2) = 3$, what is the value of $g(4)$?

(A) −0.020 (B) 0.152 (C) 3.031 (D) 3.152

9. (calculator not allowed)

Which of the following is an equation of the line tangent to the graph of $y = 2 - \int_1^x e^{t^2}\,dt$ at the point where $x = 1$?

(A) $y = 2$ (B) $y = 2(x-1) - e$ (C) $y = -e(x-1) + 2$ (D) $y = -2e(x-1) + 2$

10. (calculator not allowed)

If $f(x) = \int_1^{x^3} \frac{1}{1+\ln t}\,dt$ for $x \geq 1$, then $f'(2) =$

(A) $\frac{1}{1+\ln 2}$ (B) $\frac{12}{1+\ln 2}$ (C) $\frac{1}{1+\ln 8}$ (D) $\frac{12}{1+\ln 8}$

11. (calculator allowed)

If $f'(x) = \ln(x^2)$ and $f(5) = 8$, then $f(3) =$

(A) 2.497 (B) 4.171 (C) 5.502 (D) 13.502

12. (calculator allowed)

Let g be the function given by $g(x) = \int_0^x \sin(t^2)dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

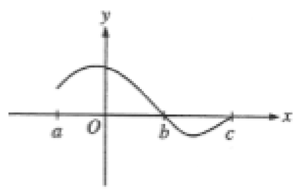
(A) $-1 \le x \le 0$ (B) $0 \le x \le 1.772$ (C) $1.253 \le x \le 2.171$ (D) $1.772 \le x \le 2.507$ (E) $2.802 \le x \le 3$

13. (calculator allowed)

If $0 \le x \le 4$, of the following, which is the greatest value of x such that $\int_0^x (t^2 - 2t)\, dt \ge \int_2^x t\, dt$?

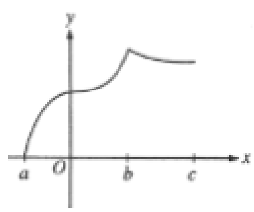
(A) 1.35 (B) 1.38 (C) 1.41 (D) 1.48 (E) 1.59

14. (calculator allowed)

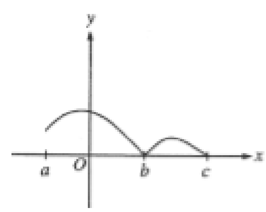


Let $f(x) = \int_a^x h(t)\, dt$, where h has the graph shown above. Which of the following could be the graph of f?

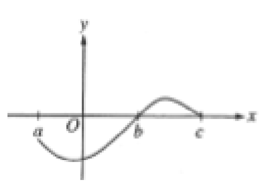
(A)



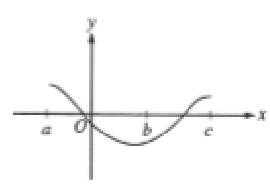
(B)



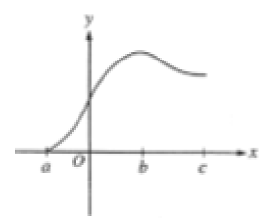
(C)



(D)



(E)



15. (calculator allowed)

A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F (B) 119°F (C) 147°F (D) 238°F (E) 335°F

16. (calculator allowed)

For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x)=\int_2^x f(t)\,dt$. Which of the following could be a table of values for g?

(A)

x	$g(x)$
1	–2
2	0
3	1

(B)

x	$g(x)$
1	–2
2	0
3	3

(C)

x	$g(x)$
1	1
2	0
3	–2

(D)

x	$g(x)$
1	2
2	0
3	–1

(E)

x	$g(x)$
1	3
2	0
3	2

17. (calculator allowed)

The rate of change of the altitude of a hot-air balloon is given by $r(t)=t^3-4t^2+6$ for $0\le t\le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

(A) $\int_{1.572}^{3.514} r(t)\,dt$ (B) $\int_0^8 r(t)\,dt$ (C) $\int_0^{2.667} r(t)\,dt$ (D) $\int_{1.572}^{3.514} r'(t)\,dt$ (E) $\int_0^{2.667} r'(t)\,dt$

Fundamental Theorem of Calculus Friday – 6 April 2018 IN CLASS WORK FRQ #18-20

Complete work for all questions on a separate sheet of paper.

Free Response

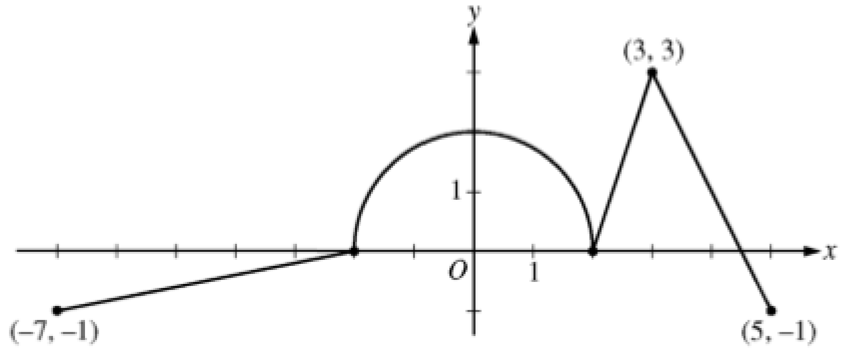
18. (calculator not allowed)

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2\le x\le 13$.

(b) Evaluate $\int_2^{13}(3-5f'(x))\,dx$. Show the work that leads to your answer.

19. (calculator not allowed)



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

20. (calculator allowed)
As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

(c) Evaluate $\int_0^{10} H'(t)\, dt$. Using correct units, explain the meaning of the expression in the context of this problem

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

Fundamental Theorem of Calculus Friday – 6 April 2018 HOMEWORK FRQ #21-24

Complete work for all questions on a separate sheet of paper.

21. (calculator allowed)

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

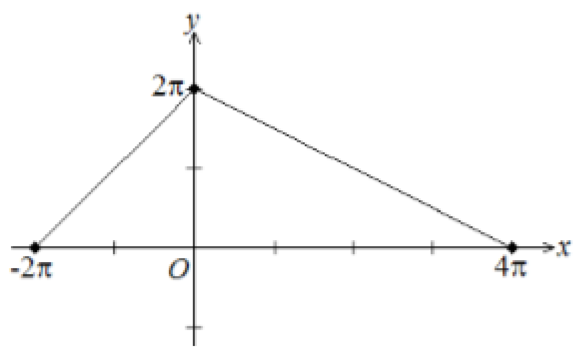
$$\frac{dR}{dt} = \frac{1}{16}\left(3+\sin(t^2)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

22. (calculator not allowed)

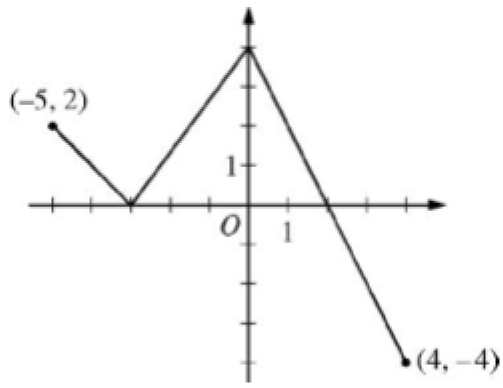


Let g be the piecewise-linear defined function on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

23. (calculator not allowed)



Graph of f

The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x)=\int_{-3}^{x} f(t)\,dt.$

(b) On what open intervals contained in $-5<x<4$ is the graph of g both increasing and concave down? Give a reason for your answer.

24. (calculator not allowed)
The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t)=20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0\le t\le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t)=-0.04t^3+0.4t^2+0.96t$ cubic feet per hour, for $0\le t\le 8$. There are 30 cubic feet of water in the pipe at time $t=0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0\le t\le 8$?

(c) At what time $0\le t\le 8$, is the amount of the pipe at a minimum? Justify your answer.