Motion with Integrals
Worksheet 4: What you need to know about Motion along the x-axis (Part 2)

1. Speed is the absolute value of ________________________.

2. If the velocity and acceleration have the ________________ sign (either both positive or both negative), then speed is ______________________________.

3. If the velocity and acceleration have the ________________ sign (one positive and one negative), then speed is ______________________________.

There are three ways to use an integral in the study of motion that are easily confused. Watch out!

4. $\int v(t)\,dt$ is an _________________________ integral. It will give you an expression for _________________________ at time t. Don't forget that you will have a _____________, the value of which can be determined if you know a position value at a particular time.

5. $\int_{t1}^{t2} v(t)\,dt$ is a ____________________ integral and so the answer will be a ___________. The numerical value represents the change in ________________________ over the time interval from _______ to _________. By the Fundamental Theorem of Calculus, since $v(t) = x'(t)$, the integral will yield ____________. This is also known as displacement. The answer can be positive or negative depending on if the particle lands to the ____________ or to the _________ of its original __________________ position.

6. $\int_{t1}^{t2} |v(t)|\,dt$ is another example of a ____________________ integral and so the answer will be a ____________. The numerical value represents the _________________________ traveled by the particle over the time interval from ___________ to _________. The answer should always be ______________________.

More Reasoning with Tabular Data

4. The rate at which water is being pumped into a tank is given by the continuous, increasing function, $R(t)$. A table of selected valued of $R(t)$, for the time interval $0 \le t \le 20$ minutes, is shown above.

t (min)	0	4	9	17	20
$R(t)$ (gal/min)	25	28	33	42	46

a. Use a right Riemann sum with four subintervals to approximate the value of $\int_0^{20} R(t)\,dt$.

Is your approximation greater of less than the true value? Give a reason for your answer.

b. A model for the rate at which the water is being pumped into the tank is given by the function: $W(t) = 25e^{0.03t}$, where t is measured in minutes and $W(t)$, is measures in gallons per minute. Use the model to find the average rate at which water is being pumped into the tank from time $t = 0$ to $t = 20$ minutes.

c. The tank contained 100 gallons of water at time $t = 0$ minutes. Use the model in part (b) to find the amount of water in the tank at $t = 20$ minutes.

5. Car A has a positive velocity $V_A(t)$ as it travels on a straight road, where V_A is measured in (feet/sec) is a differentiable function of time t in (seconds). The velocity over the time interval $0 \le t \le 10$ seconds is shown in the table above.

t (sec)	0	2	5	7	10
$V_A(t)$ (ft/sec)	0	9	36	61	115

a. Use the data in the table to approximate the acceleration of Car A at $t = 8$ seconds. Indicate units of measure.

b. Use data from the table to approximate the distance traveled by Car A over the interval $0 \le t \le 10$ seconds by using a trapezoidal sum with four subintervals. Show the computations that lead to your answer and indicate units of measure.

(#5 - continued)

c. Car B travels along the same road with an acceleration of $a_B(t) = 2t + 2\ \left(ft/\sec^2\right)$. At time $t = 3$ seconds, the velocity of Car B is 11 ft/sec. Which car is traveling faster at time $t = 7$ seconds? Explain your answer.

6. A particle moves along a horizontal line with a positive velocity $v(t)$, where is measured in (cm/sec) is a differentiable function of time t in (seconds). The velocity of the particle at selected times is given in the table above.

t (sec)	0	2	4	6	8	10	12
$v(t)$ $(cm/\sec)$	37	17	5	1	6	17	38

a. Based on the values in the table, what is the smallest number of times at which the velocity pf the particle could equal 20 cm/sec in the open interval $0 < t < 12$ seconds? Justify your answer.

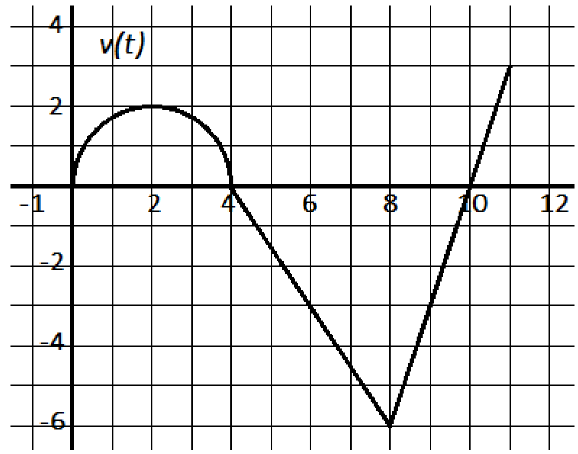
b. Based on the values in the table, what is the smallest number of times at which the acceleration of the particle could equal zero in the open interval $0 < t < 12$ seconds? Justify your answer.

c. Find the average acceleration of the particle over the time interval $8 < t < 10$ seconds? Show the computations that lead to your answer and indicate units of measure.

d. Use a midpoint Riemann sum with three subintervals of equal length and values from the table to approximate: $\int_0^{12} v(t)dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of this definite integral in terms of the particle's motion.

Worksheet 5: Sample Practice Problems for the Topic of Motion. (Part 2)

Example 1 (graphical)



The graph to the right shows the velocity $v(t)$, of a particle moving along the x-axis for $0 \le t \le 11$. It consists of a semi-circle and two line segments. Use the graph and your knowledge of motion to answer the following questions.

1. At what time t on $0 \le t \le 11$ is the speed of the particle the greatest?

2. At which of the times $t = 2,\ t = 6,\ or\ t = 9$ is the acceleration of the particle the greatest? Explain your answer.

3. Over what time intervals is the particle moving to the left? Explain your answer.

4. Over what time intervals is the speed of the particle decreasing? Explain your answer.

5. Find the total distance traveled by the particle over the time interval $0 \le t \le 11$.

6. Find the value of $\int_0^{11} v(t)\,dt$ and explain the meaning of this integral in the context of the problem.

7. If at time $t = 0$, the particle's initial position is $x(0) = 2$, complete the equation for the position of the particle at time $t = 11$.

$x(11) =$ ______________________________

Example 2 (analytical/graphical/calculator active)

The rate of change, in kilometers per hour, of the altitude of a hot air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 4$, where t is measured in hours. Assume the balloon is initially at ground level.

1. For wat values of t, $0 \le t \le 4$, is the altitude of the balloon decreasing? Justify your answer.

2. Find the values of $r'(2)$ and explain the meaning of the answer in the context of the problem. Indicate units of measure.

3. What is the altitude of the balloon when it is closest to the ground during the time interval, $2 \le t \le 4$?

4. Find the value of $\int_0^4 r(t)\,dt$ and explain the meaning of the answer in the context of the problem. Indicate units of measure.

5. Find the value of $\int_0^4 \left|r(t)\right| dt$ and explain the meaning of the answer in the context of the problem. Indicate units of measure.

6. What is the maximum altitude of the balloon during the time interval $0 \le t \le 4$?

Example 3 (numerical)

time t	0	2	6	10
velocity $v(t)$	5	3	–1	–8
acceleration $a(t)$	0	–1	–3	–5

The table gives the values for the velocity and acceleration of a particle moving along the *x*-axis for selected values of time t. Both velocity and acceleration are differentiable functions of time t. The velocity is decreasing for all values of t, $0 \le t \le 10$. Use the data in the table to answer the questions that follow.

1. Is there a time t when the particle is at rest? Explain your answer.

2. At what time indicated in the table is the speed of the particle decreasing? Explain your answer.

3. Use a left Riemann sum to approximate $\int_0^{10} v(t)\,dt$. Show the computations that lead to your answer. Explain the meaning of the definite integral in the context of the problem.

4. Is the approximation in part (3) greater or less than the actual value of the definite integral? Explain you reasoning.

5. Approximate the value of $\int_0^{10} |v(t)|\,dt$ using a trapezoidal approximation with 3-subintervals indicated by the values in the table. Show the computations that lead to your answer. Explain the meaning of the definite integral in the context of the problem.

6. Determine the value of $\int_0^{10} a(t)\,dt$. Explain the meaning of the definite integral in the context of the problem.

FRQ **Calculator Active**

A particle moves along the y-axis so that it's velocity, $v(t)$ at time $t \geq 0$ is given by $v(t) = 1 - \arctan\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$.

1. Find the acceleration of the particle at $t = 2$.

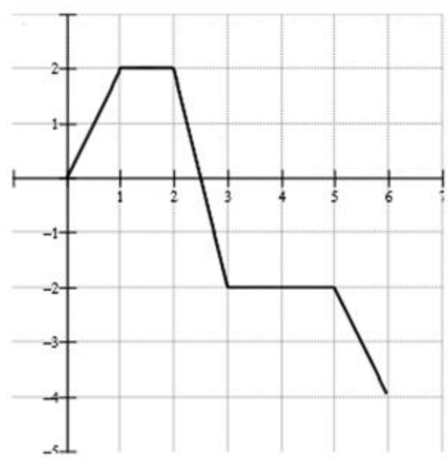
2. Is the speed of the particle increasing or decreasing at time, $t = 2$? Justify your answer.

3. Find the time, $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

4. Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at $t = 2$? Justify your answer.

FRQ - Motion **Non-Calculator**

Use the graph of velocity $v(t)\ in\ (m/\sec)$ for a particle moving along the *x*-axis on $0 \leq t \leq 6$ to complete the following questions and suppose that $x(0) = -1$.



a. Complete the table.

t	0	1	2	3	4	5	6
$x(t)$							

b. Make a horizontal motion graph for the particle.

c. Use a colored pencil to draw the speed graph on $0 \leq t \leq 6$.

d. When is the particle moving left? Moving right? Justify.

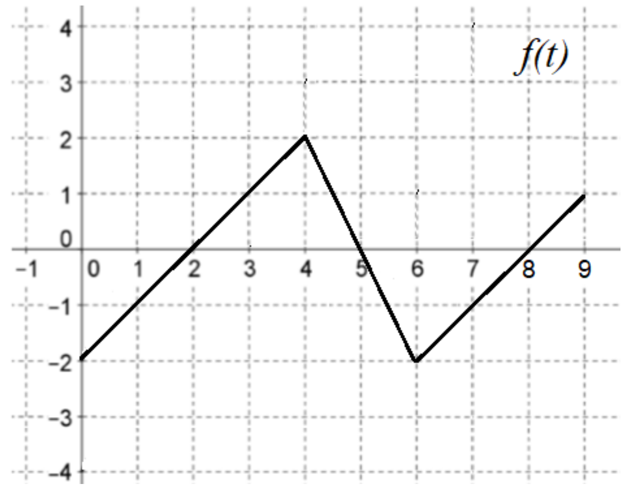
e. When is the particle speeding up? Slowing down? Justify

f. Write an integral that represents the particles total distance traveled. Evaluate using correct units.

g. Write an equation using an integral that represents the position of the particle at time $t = 6$ sec. Evaluate & find $x(6)$.

h. If the particle the particles position is $s(t)$ representing a height over time, not a position along the x-axis, then draw the position function on the same graph as the velocity. Include accurate concavity behavior.

FRQ – Integral Defined Functions



Use the figure at the right, sketch the graph of these function

a. $F(x)=\int_0^x f(t)\,dt$

b. $G(x)=\int_2^x f(t)\,dt$

Complete the table for each function.

x	0	1	2	3	4	5	6	7	8	9
$F(x)$										
$G(x)$										

1. On what intervals are these functions

   a. increasing? Justify.

   b. decreasing? Justify.

   c. concave up? Justify.

   d. concave down? Justify.

2. For what values of *x* do each of these functions have

   a. a maximum? Justify.

   b. a minimum? Justify.

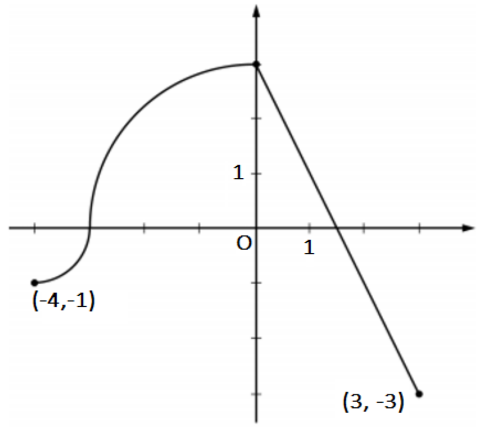
   c. point of inflection? Justify

3. For what values of *x* do each of these functions have

   a. an absolute maximum? Justify.

   b. an absolute minimum? Justify.

LT: I can read and interpret an integral defined function.



Graph of f

5. Let f be the continuous function defined on [-4, 3]. The graph of f consists of two quarter circles and one line segment as shown in the figure above. Let g be the function given by $g(x) = \int_0^x f(t)dt$.

(a) Find the values of $g(-3)$, $g(1)$ and $g(3)$.

$g(-3) =$ ___________ $g(1) =$ ___________ $g(3) =$ ___________

(b) On what interval(s) is $g(x)$ decreasing and concave up?

(c) Where does g have a local maximum value? Justify your answer.

(d) Where does g have a local minimum value? Justify your answer.

(e) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(f) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

2013 AP FRQ #2 **Calculator Active**

A particle moves along a straight line. For time, $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{\frac{6}{5}} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

1. Find all the values of t on the interval $2 \le t \le 4$, for which the speed of the particle is 2.

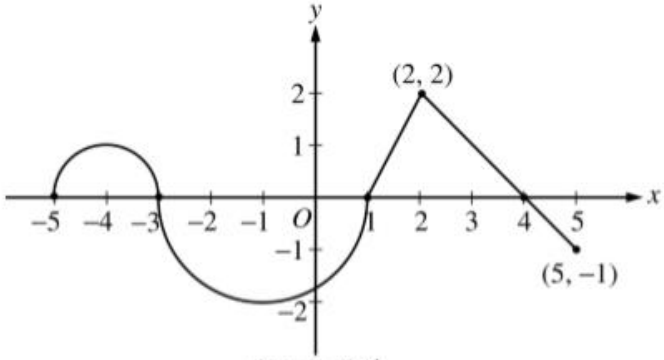
2. Write an expression involving an integral that gives the position, $s(t)$. Use this expression to find the position of the particle at time, $t = 5$.

3. Find all times, t, on the interval $0 \le t \le 5$, at which the particle changes direction. Justify your answer.

4. Is the speed of the particle increasing or decreasing at time, $t = 4$? Justify your answer.

5. When is the particle furthest from the origin on the time interval $0 \le t \le 5$? Show the work that leads to your answer.

2007B AP FRQ #4 **Non-Calculator**



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

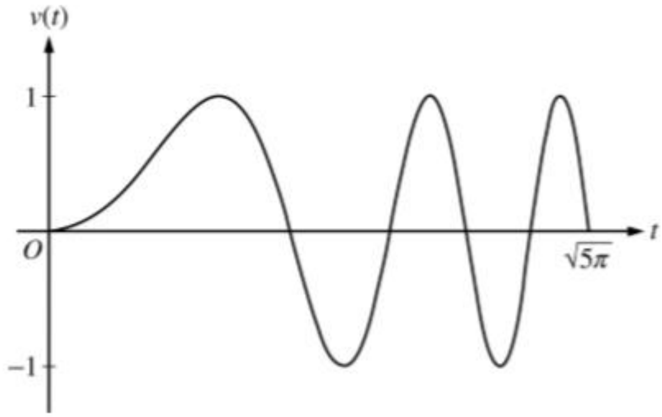
(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

2007B AP FRQ #2 **Calculator Active**



2. A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.