

TWO TRIG LIMITS EXPLORATION: Let's examine the following limit using technology to help us evaluate.

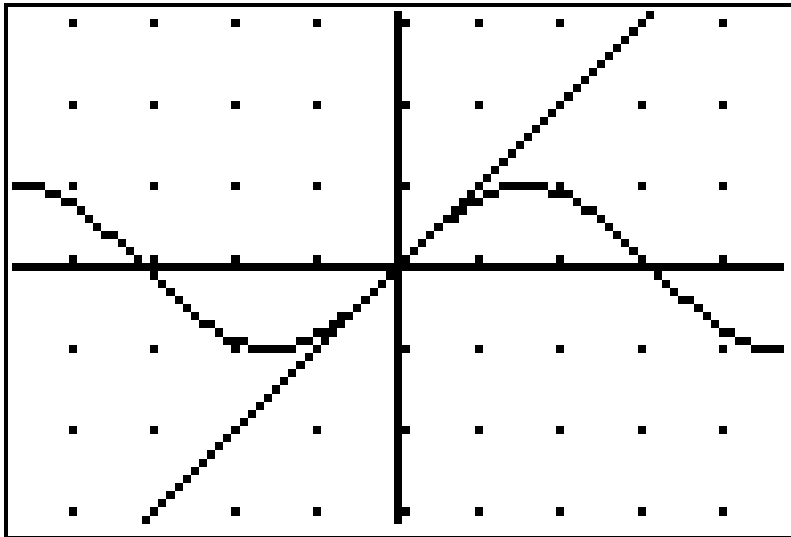
$$\#1) \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = ?$$

Let's look at the graphs of the numerator and denominator separately, on the same graph. On your graphing calculator, set the following 1) Y= 2) MODE 3) ZOOM to obtain the graph.

```
Plot1 Plot2 Plot3
Y1=sin(X)
Y2=X
Y3=
Y4=
Y5=
Y6=
Y7=
```

```
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
↓NEXT↓
```

```
ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7↓ZTrig
```



Notice that very close to zero the graph of $y=\sin(x)$ and $y=x$ look very much the same.

So what value would you assign to

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = ?$$

In Y= turn off Y1 and Y2. Enter Y3 as shown. GRAPH and evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = ?$

```
Plot1 Plot2 Plot3
Y1=sin(X)
Y2=X
Y3=Y1/Y2
Y4=
Y5=
Y6=
Y7=
```



TRACE to $x = 0$. Move the left and right arrow keys to the left and right of $x=0$.

What appears to be happening at this x-value?

Press 2nd ZOOM to turn off the axes and GRAPH again. Now what do you see, or don't you see?

Let's examine the TABLE before we conclude. Press 2nd WINDOW to get to the TABLE SETUP menu. Set it as shown. Press 2nd GRAPH to see the table and use the up arrow to scroll to $x = -3$.

```
TABLE SETUP
TblStart=0
ΔTbl=1
Indent: Auto Ask
Depend: Auto Ask
```

X	Y3
-3	.04704
-2	.45465
-1	.84147
0	ERROR
1	.84147
2	.45465
3	.04704

X = -3

Examine the y-values in the table as x gets closer to zero from the left and as x gets closer to zero from the right.

What appears to be happening?

Return to the Table SetUp. Reset the TblStart =0. Now let's zoom in on the table by changing the Table Step shown as ΔTbl from 1 to 0.01.

```
TABLE SETUP
TblStart=0
ΔTbl=.01
Indent: Auto Ask
Depend: Auto Ask
```

Press 2nd GRAPH to view the table again.

Examine the y-values in the table as x gets closer to zero from the left and as x gets closer to zero from the right.

What appears to be happening?

X	Y3
-.03	.99985
-.02	.99993
-.01	.99998
0	ERROR
.01	.99998
.02	.99993
.03	.99985

X = -.03

In conclusion, we now know graphically that

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = \underline{\hspace{2cm}}$$

Now let's examine the limits of variations of #1. Use your calculator to determine the values of these limits. All answers must be exact, no decimals. So use fraction form!

a) $\lim_{x \rightarrow 0} \left(\frac{\sin(8x)}{8x} \right) = ?$

b) $\lim_{x \rightarrow 0} \left(\frac{\sin(2x)}{2x} \right) = ?$

c) $\lim_{x \rightarrow 0} \left(\frac{\sin(x)}{6x} \right) = ?$

d) $\lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{x} \right) = ?$

e) $\lim_{x \rightarrow 0} \left(\frac{\sin(9x)}{4x} \right) = ?$

f) $\lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{7x} \right) = ?$

PRACTICE: Evaluate the following limits. DO NOT USE your calculator.

G) $\lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{5x} \right) = ?$

H) $\lim_{x \rightarrow 0} \left(\frac{\sin(7x)}{2x} \right) = ?$

I) $\lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{6x} \right) = ?$

J) $\lim_{x \rightarrow 0} \left(\frac{9 \sin(x)}{2x} \right) = ?$

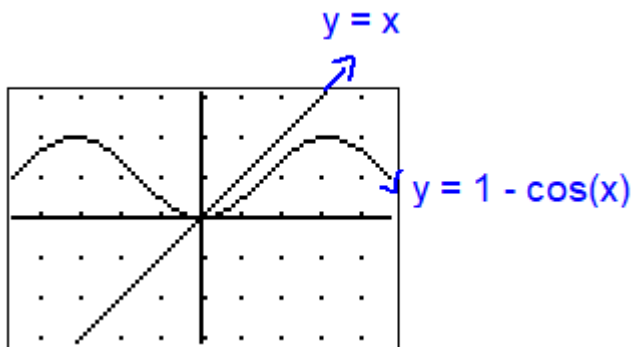
K) $\lim_{x \rightarrow 0} \left(\frac{\sin(12x)}{4x} \right) = ?$

L) $\lim_{x \rightarrow 0} \left(\frac{\sin(15x)}{10x} \right) = ?$

Let's examine the following limit using technology to help us evaluate.

#2) $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x} \right) = \underline{\hspace{2cm}} ?$

After graphing $Y1 = 1 - \cos(x)$ and $Y2 = x$, what are your thoughts about this limit?



Let's take a closer look using the table.

We'll look at the table near $x=0$ with a two different ΔTbl zooming in toward zero.

- 1) $\Delta Tbl = 0.01$ 2) $\Delta Tbl = 0.0001$

For $\Delta Tbl = 0.01$, examine the LHL and the RHL.

$$\lim_{x \rightarrow 0^-} \frac{1 - \cos(x)}{x} \Rightarrow \left. \frac{1 - \cos(x)}{x} \right|_{x=-0.01} = \frac{+0.}{-0.} = \underline{\hspace{2cm}} \dots$$

$$\lim_{x \rightarrow 0^+} \frac{1 - \cos(x)}{x} \Rightarrow \left. \frac{1 - \cos(x)}{x} \right|_{x=+0.01} = \frac{+0.}{+0.} = \underline{\hspace{2cm}} \dots$$

For $\Delta Tbl = 0.0001$, examine the LHL and the RHL.

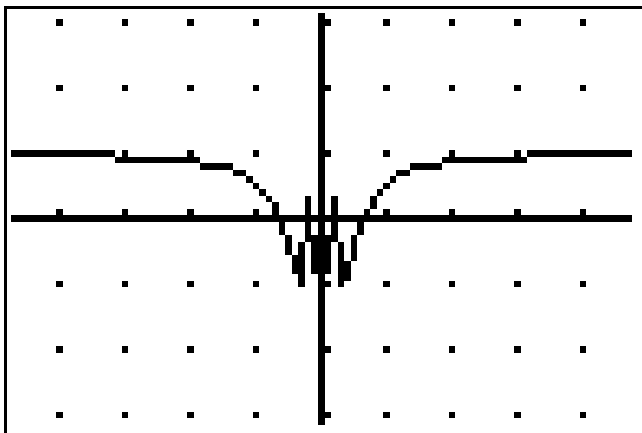
$$\lim_{x \rightarrow 0^-} \frac{1 - \cos(x)}{x} \Rightarrow \left. \frac{1 - \cos(x)}{x} \right|_{x=-0.0001} = \frac{+0.}{-0.} = \underline{\hspace{2cm}} \dots$$

$$\lim_{x \rightarrow 0^+} \frac{1 - \cos(x)}{x} \Rightarrow \left. \frac{1 - \cos(x)}{x} \right|_{x=+0.0001} = \frac{+0.}{-0.} = \underline{\hspace{2cm}} \dots$$

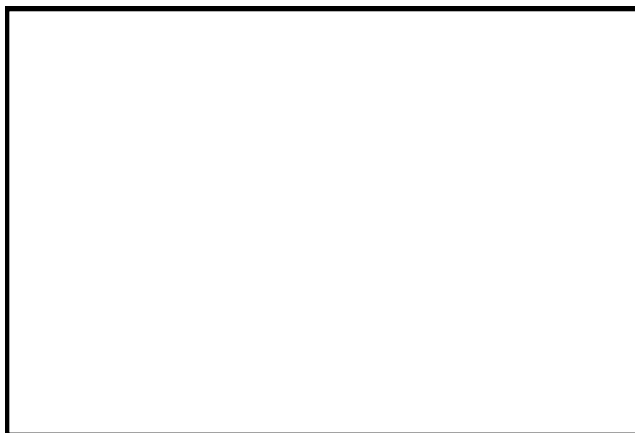
After closer examination of the left and right-hand limits as x approaches zero, the limit values appear to be getting closer and closer to what value?

In conclusion: $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x} \right) = \underline{\hspace{2cm}}$

Explore the behavior of the graph $f(x) = \cos\left(\frac{1}{x}\right)$ near $x = 0$ by using the following windows to zoom in near zero.



X: [-4.7, 4.7] Y: [-3.1, 3.1]



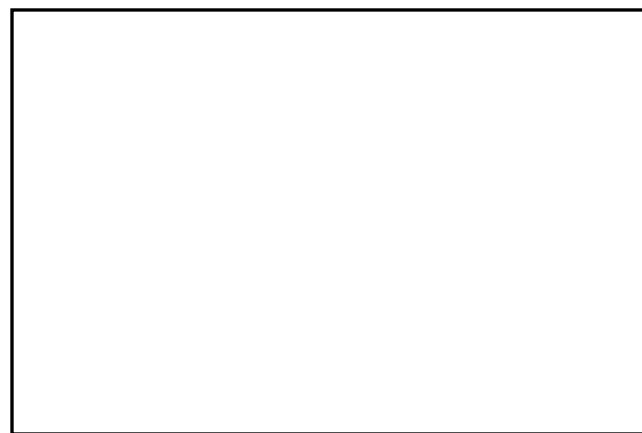
X: [-2, 2] Y: [-1.2, 1.2]



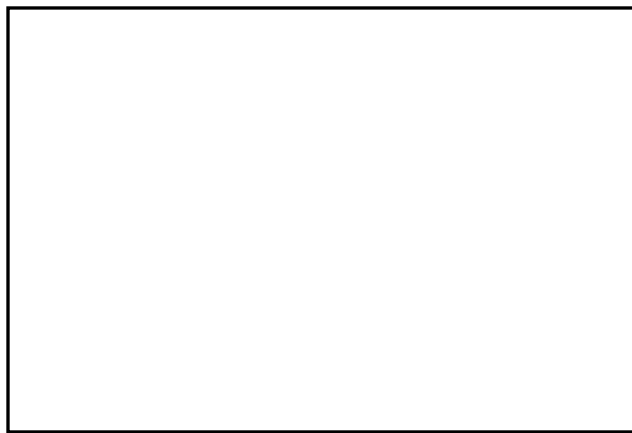
X: [-1, 1] Y: [-1.2, 1.2]



X: [-0.5, 0.5] Y: [-1.2, 1.2]



X: [-0.25, 0.25] Y: [-1.2, 1.2]



X: [-0.10, 0.10] Y: [-1.2, 1.2]

Repeat to explore the behavior of the graph $f(x) = \sin\left(\frac{1}{x}\right)$ near $x = 0$. Pg4