ANSWER KEY

§3.1 - §3.6 Applying the Derivative Rules using Tables

The purpose of this worksheet is to abstract the concept of the derivative rules by causing you to apply them to functions that you do not know. Two functions, $f(x)$ and $g(x)$, have the values and first derivatives shown in the table. Use this information to find the following.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-4	2	-2	-1	1
-3	1	-1	-2	2
-2	-2	1	0	3
-1	-1	4	2	1
0	0	5	1	0
1	2	3	0	-2
2	3	2	1	-1
3	3	1	-1	-3
4	1	-1	-2	-4

1. $h(x) = f(x) - g(x)$
   Find $h'(2)$
   $h'(x) = f'(x) - g'(x)$
   $h'(2) = f'(2) - g'(2)$
   $= 1 - (-1)$
   $= 2$

2. $h(x) = f(x) + 3g(x)$
   Find $h'(0)$
   $h'(x) = f'(x) + 3g'(x)$
   $h'(0) = f'(0) + 3g'(0)$
   $= 1 + 3(0)$
   $= 1$

3. $h(x) = 2f(x) - 4g(x)$
   Find $h'(-3)$
   $h'(-3) = 2f'(-3) - 4g'(-3)$
   $= 2(-2) - 4(2)$
   $= -12$

4. $h(x) = 2f(x) - 1$
   Find $h'(3)$
   $h'(3) = 2f'(3) - 1$
   $= 2(-1) - 1$
   $= -3$

5. $h(x) = 3g(x) - x^2$
   Find $h'(1)$
   $h'(1) = 3g'(1) - 2(1)$
   $= 3(-2) - 2$
   $= -8$

6. $h(x) = x\,f(x)$
   Find $h'(-1)$
   $h'(x) = 1 \cdot f(x) + x \cdot f'(x)$
   $h'(-1) = 1\,f(-1) - 1\,f'(-1)$
   $h'(-1) = 1(-1) - 1(2)$
   $h'(-1) = -3$

7. $h(x) = [f(x)]^2$ $h'(x) = 2 \cdot f(x) \cdot f'(x)$
   Find $h'(-3)$
   $h'(-3) = 2f(-3) \cdot f'(-3)$
   $= 2(1)(-2)$
   $= -4$

8. $h(x) = f(x)g(x)$
   Find $h'(2)$ $f'(x)g(x) + f(x)g'(x)$
   $h'(2) = f'(2) \cdot g(2) + f(2)g'(2)$
   $= (1)(2) + (3)(-1)$
   $= 2 - 3$
   $= -1$

9. $h(x) = x^2 \boxed{f(x)g(x)}$
   Find $h'(-1)$ (Use #8 ANSWER)
   $h'(x) = 2x(f \cdot g) + x^2(f' \cdot g + f \cdot g')$
   $h'(-1) = 2(-1)(-1)(4) + (1)(2(4) + (-1) \cdot 1)$
   $h'(-1) = 8 + 7 = 15$

10. $h(x) = f(x)/g(x)$
    Find $h'(-2)$ $h' = \dfrac{f'g - fg'}{g^2}$
    $h'(-2) = \dfrac{0 - (-6)}{1^2}$
    $h'(-2) = 6$

11. $h(x) = f(3x)$
    Find $h'(-1)$
    $h'(x) = f'(3x)(3)$
    $h'(-1) = f'(-3) \cdot 3$
    $= (-2)(3)$
    $= 6$

12. $h(x) = g\left(x^2\right)$
    Find $h'(-2)$
    $h'(x) = g'(x^2) \cdot (2x)$
    $h'(-2) = g'(4)(-4)$
    $= (-4)(-4)$
    $= 16$

13. $h(x) = f\left(x^3 - x\right)$
    Find $h'(1)$
    $h'(x) = f'(x^3 - x)(3x^2 - 1)$
    $h'(1) = f'(0) \cdot (2)$
    $= (1)(2)$
    $= 2$

14. $h(x) = f(g(x))$
    Find $h'(4)$
    $h'(x) = f'(g(x)) \cdot g'(x)$
    $h'(4) = f'(g(4)) \cdot g'(4)$
    $= f'(-1) \cdot g'(4) = (2)(-4)$
    $= -8$

15. $h(x) = g(f(x))$
    Find $h'(-3)$
    $h'(x) = g'(f(x)) \cdot f'(x)$
    $h'(-3) = g'(f(-3))\ f'(-3)$
    $= g'(1) \cdot f'(3)$
    $= (-2)(-1) = 2$

16. $h(x) = [f(x)]^3 \cdot g(-2x)$
    Find $h'(2)$
    $h'(x) = 3(f(x))^2 \cdot f'(x)\,g(-2x) + (f(x))^3 \cdot g'(-2x)(-2)$
    $h'(2) = 3(3)^2 \cdot (1) \cdot (-2) + (3)^3 \cdot (1)(-2)$
    $= -54 - 54 = -108$

17. $h(x) = x^2/f(x)$ $h'(x) = \dfrac{f(x) \cdot 2x - x^2 \cdot f'(x)}{(f(x))^2}$
    Find $h'(-1)$
    $h'(-1) = \dfrac{(-1)(-2) - (1)(2)}{1}$
    $h'(-1) = \dfrac{2-2}{1} = 0$

18. $h(x) = f(\ln x)/g(2x+1)$
    Find $h'(1)$ $h'(x) = \left(\frac{1}{x}\right) f'(\ln x)\, g(2x+1) - f(\ln x) \cdot g'(2x+1)(2)$
    $h'(1) = 1 \cdot f'(0) \cdot g(3) - f(0) \cdot g'(3) \cdot 2$
    $h'(1) = (1)(1) - (0)(-3) \cdot 2$
    $= 1 - 0$
    $= 1$

ANSWER KEY

$\boxed{\#}$ 3 decimal place accuracy

UNIT 2 Concept of Derivative **FOCUS: Smart use of technology for AROC & IROC calculations**

	ON YOUR PAPER	***ON CALCULATOR***
1 Given the function $f(x)$, **show the calculation** necessary to find the: a) <u>average rate of change</u>, <u>average velocity</u>, or <u>slope of the secant</u> on the interval $x \in [4.3, 5.6]$ ....>	$\dfrac{f(5.6)-f(4.3)}{5.6-4.3} = \boxed{\#}$	$\dfrac{(Y_1(5.6)-Y_1(4.3))}{(5.6-4.3)} = \boxed{\#}$
b) <u>instantaneous rate of change</u>, <u>instantaneous velocity</u>, or <u>slope of the tangent</u> line at $x = 4.95$ ....>	$\left.\dfrac{d(f(x))}{dx}\right\vert_{x=4.95} = \boxed{\#}$	$\left.\dfrac{d(Y_1)}{dx}\right\vert_{x=4.95} = \boxed{\#}$

2 The height of a projectile propelled from a platform <u>120 feet</u> (h_0) in the air with an initial velocity <u>of 96 ft/sec</u> (v_0) is given by the function $h(t) = -\frac{1}{2}a_0t^2 + v_0t + h_0$. Note: Earth's gravitational constant is ($a_0=$) <u>32 ft/sec²</u>.

Write the equation for $h(t)$ = $-16t^2+96t+120$ and **show the calculation** necessary to find the:

a) <u>average rate of change</u>, <u>average velocity</u>, or <u>slope of the secant</u> on each of the intervals

Interval	$t \in [0,1]$	$t \in [1,2]$	$t \in [2,3]$	$t \in [3.012, 5.789]$	$t \in [4.218, 6.357]$
Algebraic Expression in terms of h(t)	$\frac{h(1)-h(0)}{1-0}$	$\frac{h(2)-h(1)}{2-1}$	$\frac{h(3)-h(2)}{3-2}$	$\frac{h(5.789)-h(3.012)}{5.789-3.012}$	$\frac{h(6.357)-h(4.218)}{6.357-4.218}$
Evaluation (3-decimal accuracy)	$= 80 \frac{ft}{sec}$	$= 48 \frac{ft}{sec}$	$= 16 \frac{ft}{sec}$	$= -44.816 \frac{ft}{sec}$	$= -73.2 \frac{ft}{sec}$

* *Since function is defined, write the expression for the slope of secant and evaluate the expression on the calculator. Record 3-decimal accuracy.*

WITH UNITS!

b) <u>instantaneous rate of change</u>, <u>instantaneous velocity</u>, or <u>slope of the tangent</u> line at $t = 3.724$ seconds..........> $\boxed{\left.\dfrac{d(h(t))}{dt}\right\vert_{t=3.724} \approx -23.168 \frac{ft}{sec}}$ Let the calculator do the work

Record calculation on paper

c) Examine the values for the first three intervals what do they tell you about the behavior of the function. You should be able to conclude two specific ideas. → 80 ft/sec to 48 ft/sec to 16 ft/sec

$h'(t) > 0$ for $t \in (0,3)$ so $h(t)$ is increasing or ht of projectile is increasing.
but the height is increasing at a decreasing rate. Graph of $h(t)$ is inc & ccd. Like this:

3 Given the table of values

$x_{(sec)}$	0	1	2	3	4	5	6	7
$f(x)_{(meters)}$	120	200	248	264	248	200	120	8

show the calculation necessary to find the: $f'(x)$ UNITS: $\frac{meters}{sec}$

***ON PAPER:** must pull values from the table and use in calculation*

a) <u>average rate of change</u>, <u>average velocity</u>, or <u>slope of the secant</u> on the interval $x \in [4,6]$ ....> $\dfrac{f(6)-f(4)}{6-4} = \dfrac{120-248}{6-4} = -64 \frac{m}{sec}$

b) <u>instantaneous rate of change</u>, <u>instantaneous velocity</u>, or <u>slope of the tangent</u> line at $x = 5$ ....> $\left.\dfrac{d(f(x))}{dx}\right\vert_{x=5} \approx \dfrac{120-248}{6-4} = -64 \frac{m}{sec}$

Must use data in table. to estimate $f'(5)$

Using appropriate **MATHEMATICAL NOTATION** to write what is required to justify Continuity & Differentiability.

4 Definition of Continuity in 3 parts.

a) $f(c) = L$ (means $f(c)$ exists)

b) $\lim\limits_{x \to c} f(x) = L$ (means LHL = RHL)

c) $f(c) = \lim\limits_{x \to c} f(x) = L$ (means both the y-value & the limit value both equal L)

" NOTATION MATTERS LEARN IT & ALWAYS USE IT CORRECTLY! "

5 Definition of Differentibility.

$f'(c^-) = f'(c^+)$

$\lim\limits_{x \to c^-} f'(x) = \lim\limits_{x \to c^+} f'(c)$

means slope on left side of x=c EQUALS slope on right side of x=c

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