Mean Value Theorem, Extreme Value Theorem, Intermediate Value Theorem and Related Rate Questions

2003 #80

The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$.
(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.
(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$.
(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.
(E) There exists c, where $-2 \le x \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

2008 #89

The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.
(B) For $-2 < k < 2$, $f'(k) < 0$.
(C) For $-2 < k < 2$, $f'(k)$ exists.
(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.
(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

2008 #88

The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π