p. 617–618 #7, 8, 10, 15, 19, 21, 23, 26

DAY 135

7) (t, T)
$(0, 20)$

a) $\frac{dH}{dt} = -k(H - 200)$

$\int \frac{dH}{H-200} = \int -k\,dt$

$\ln|H-200| = -kt + c \Big|_{(0,20)}$

$\ln|-180| = c$

$c = \ln(180)$

$\ln|H-200| = -kt + \ln 180$

$H - 200 = \pm 180 e^{-kt}$

$H = -180 e^{-kt} + 200$

b) $(30, 120°)$

$120 = 180 e^{-k(30)} + 200$

$\frac{-80}{-180} = e^{-30k}$

$+\frac{4}{9} = e^{-30k}$

$\ln\left(\frac{4}{9}\right) = -30k$

$k = -\frac{1}{30} \cdot \ln\left(\frac{4}{9}\right) \approx \boxed{+.0270}$

8) $\frac{dy}{dt} = 0.5y - 250$

a) $\frac{dy}{dt} = \frac{1}{2}(y - 500)$

EQUILIBRIUM $\frac{dy}{dt} = 0$

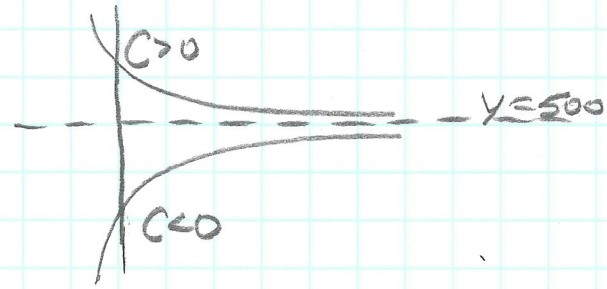
$y = 500$

b) $\frac{dy}{y-500} = \frac{1}{2}dt$

$\ln|y-500| = \frac{1}{2}t + c$

$y - 500 = \pm C e^{\frac{1}{2}t}$

$\boxed{y \pm C e^{\frac{1}{2}t} + 500}$

c) 

d) SKIP.

10) $\frac{dH}{dt} = (-k)(H - 20)$

a) The rate of change of the temperature of the cup of coffee is proportional to the difference between the current temperature of the coffee & the surrounding room temp. Because the temperature is decreasing as it cools the proportionality constant is negative. The diff eq says $(-k)$ b/c they've defined k as positive.

b) $\frac{dH}{H-20} = -k\,dt$

$\ln|H-20| = -kt + c \Big|_{(2,90)}$

$\ln(70) = -k(2) + c$

$c = 2k + \ln 70$

$H - 20 = \pm e^{-kt + 2k + \ln 70}$

$H - 20 = \pm 70 e^{-k(t-2)}$

$H = +70 e^{-k(t-2)} + 20$

Solve $60 - 20 = 70 e^{-k(t-2)}$

$\ln\left(\frac{4}{7}\right) = -k(t-2)$

p. 617-618 #15, 19, 21, 23, 26

$\boxed{\text{DAY 135}}$

(15) $\frac{dN}{dt} = kN$ $\quad (t_{hrs}, N_{mg})$

$\frac{dN}{dt} = .347N$

$\int \frac{dN}{N} = \int .347\,dt$

$\ln|N| = .347t + c$

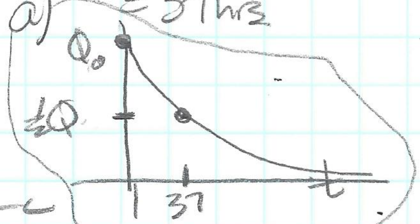
$N = Ce^{.347t}$

$C = .4$

$N = .4e^{.347t}$

(21) b) $\frac{dQ}{dt} = kQ$

a) halflife = 37 hrs



c) $\int \frac{dQ}{Q} = \int k\,dt$

$\ln|Q| = kt + c$

$Q = Ce^{kt}$ $\quad C = Q_0$ initial dose.

$\frac{Q}{C} = \frac{1}{2} = e^{37k}$

$\therefore \frac{1}{37}\ln|\tfrac{1}{2}| =$

$k = -.01873\ldots$

$Q = Ce^{-.01873t}$

$Q = .25Q_0 = .25C$

$.25\cancel{C} = \cancel{C}e^{-.01873t}$

$\ln\left(\frac{1}{4}\right) = -.01873\ldots t$

$t = \frac{\ln(.25)}{-.01873\ldots} \approx 74$ hours.

$\boxed{\text{about 3 days or 74 hours}}$

(19) $\frac{dT}{dt} = k(T - 65)$

$\int \frac{dT}{T-65} = \int k\,dt$

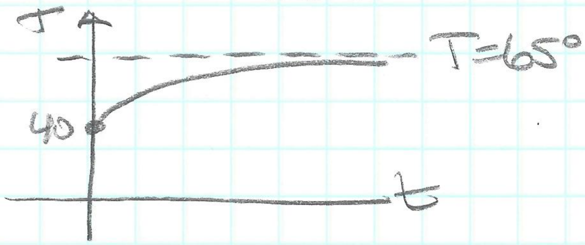
$\ln|T-65| = kt + c$

$T = Ce^{kt} + 65 \Big|_{(0,40)}$

$40 = Ce^{0} + 65$

$C = -25$

$T = -25e^{kt} + 65°$



(23) $\frac{dB}{dt} = k(B)$

a) (0, 1 million barrels of oil)
(6 yrs, 500,000 barrels) solve for k

$\int \frac{dB}{B} = \int k\,dt$

$\ln|B| = kt + c$

$B = Ce^{kt} \Big|_{(0,\ 1\text{mill})}$

$\therefore B = +1{,}000{,}000e^{kt}$

$500 = 1000e^{6k}$

$\frac{1}{2} = e^{6k}$

$\frac{1}{6}\ln|\tfrac{1}{2}| = k \approx -.115524$

$B = 1{,}000{,}000e^{-.115524t}$

→ over

PG 617-618 #23 (cont), 26 $\boxed{\text{DAY 135}}$

23 when 600,000 barrels remain $t = ?$

$600{,}000 = 1000000 e^{-.115524t}$

$.6 = e^{-.11552t}$

NOT NECESSARY TO FIND t.

$t = \frac{\ln(.6)}{-.115524...} \approx 4.421$ years there will be 600,000 barrels oil.

USE RATE TO FIND RATE

$\frac{dB}{dt} = (-.115524...)(B)\Big|_{600000} = -69{,}314 \ \frac{\text{barrels}}{\text{year}}$

So when there are 600,000 barrels of oil remaining the rate at which the amount of oil in the well is decreasing is $69{,}314 \ \frac{\text{barrels}}{\text{year}}$.

b) $50{,}000 = 1000000 e^{-.115524t}$

$0.05 = e^{-.115524t}$

$\frac{\ln|.05|}{-.115524} = t \approx \boxed{25.931 \text{ years}}$

26 9AM

$(t \quad T)$

Body Temp 9AM → $(0, 90.3°F)$ Room Temp 68°F

10AM $(1, 89.0°F)$

a) $\frac{dT}{dt} = k(T-68)$

b) $\int \frac{dT}{T-68} = \int k\, dt$

$\ln|T-68| = kt + c$

$T - 68 = Ce^{kt}$

$T = Ce^{kt} + 68$ use (I.C) solve for C

$T = 22.3e^{kt} + 68$ ← use 2nd pt solve for k

$89 - 68 = 22.3 e^{k(1)}$

$\frac{21}{22.3} = e^k$

$k = \ln\left(\frac{21}{22.3}\right) \approx -.06006$

Assume 98.6° Body Temp.

$T = 22.3e^{-.06006t} + 68$

$98.6 = 22.3e^{-.06006t} + 68$

$1.3304 = e^{-.06006t}$

$t \approx \frac{\ln(1.3304)}{-.06006} \approx -4.753$

The time the murder occurred was 4.753 hours before 9AM

$\boxed{\text{4:15AM}}$