PG 607 #22, 23, 24, 25, 26, 29, 30, 31, 32 DAY 134

(22) $\frac{dz}{dt} = te^z$ I.C $(0,0)$

$\frac{dz}{e^z} = t\,dt$

$\int e^{-z}dt = \int t\,dt$

$-e^{-z} = \frac{1}{2}t^2 + c \Big|_{(0,0)}$

$-1 = c$

$-e^{-z} = \frac{1}{2}t^2 - 1$

$e^{-z} = \frac{2-t^2}{2}$

$-z = \ln\left(\frac{2-t^2}{2}\right)$

$z = \ln\left(\frac{2}{2-t^2}\right)$

OR

$z = \ln\left(\frac{-2}{t^2-2}\right)$

(23) $\frac{dy}{dx} = \frac{5y}{x}$ I.C $(1,3)$

$\int \frac{dy}{y} = \int \frac{5}{x}dx$

$\ln|y| = 5\ln|x| + c \Big|_{(1,3)}$

$\ln|3| = c$

$\ln|y| = 5\ln|x| + \ln|3|$

$|y| = e^{5\ln|x| + \ln 3}$

$y = \pm 3x^5$ check I.C.

$y = +3x^5$

(24) $\frac{dy}{dt} = y^2(1+t)$ I.C. $(1,2)$

$\int \frac{dy}{y^2} = \int (1+t)\,dt$

$\frac{-1}{y} = t + \frac{1}{2}t^2 + c \Big|_{(1,2)}$

$-\frac{1}{2} = 1 + \frac{1}{2} + c$

$-2 = c$

$-\frac{1}{y} = t + \frac{t^2}{2} - 2$

$-\frac{1}{y} = \frac{t^2 + 2t - 4}{2}$

$y = \frac{-2}{t^2 + 2t - 4}$

(25) $\frac{dz}{dt} = z + zt^2$ I.C. $(0,5)$

$z(1+t^2)$

$\int \frac{dz}{z} = \int (1+t^2)\,dt$

$\ln|z| = t + \frac{1}{3}t^3 + c \Big|_{(0,5)}$

$\ln|5| = c$

$\ln|z| = t + \frac{1}{3}t^3 + \ln 5$

$|z| = e^{\frac{1}{3}t^3 + t + \ln 5}$

$z = \pm 5e^{\frac{1}{3}t^3 + t}$

$z = +5e^{\frac{1}{3}t^3 + t}$

PG 607 #26, 29, 30, 31, 32 — DAY 134

(26) $\frac{dw}{d\theta} = w^2 \theta \sin(\theta^2)$ (I.C. $(0,1)$)

$\int \frac{dw}{w^2} = \int \sin(\theta^2)\,\theta\,d\theta$ ← U-SUB

$-\frac{1}{w} = -\frac{1}{2}\cos(\theta^2) + c \Big|_{(0,1)}$

$-1 = -\frac{1}{2} + c$

$c = -\frac{1}{2}$

$-\frac{1}{w} = -\frac{1}{2}(\cos(\theta^2) + 1)$

$$\boxed{w = \frac{2}{\cos(\theta^2)+1}}$$

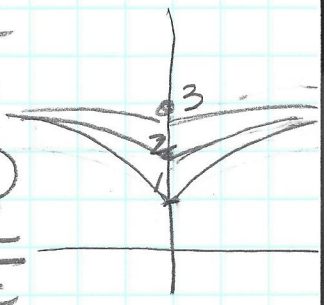
(29) a) $\frac{dy}{dx} = \frac{4x}{y^2}$

$\int y^2\,dy = \int 4x\,dx$

$\frac{1}{3}y^3 = 2x^2 + c$

$y^3 = 6x^2 + c$

$y = \sqrt[3]{6x^2 + c}$



b) $(0,1)$ $c = \frac{1}{3}$ $\quad \frac{1}{3}y^3 = 2x^2 + \frac{1}{3}$ $\quad y^3 = 6x^2 + 1$ $\quad y = \sqrt[3]{6x^2+1}$

$(0,2)$ $c = \frac{8}{3}$ $\quad \frac{1}{3}y^3 = 2x^2 + \frac{8}{3}$ $\quad y^3 = 6x^2 + 8$ $\quad y = \sqrt[3]{6x^2+8}$

$(0,3)$ $c = \frac{27}{3}$ $\quad \frac{1}{3}y^3 = 2x^2 + \frac{27}{3}$ $\quad y^3 = 6x^2 + 27$ $\quad y = \sqrt[3]{6x^2+27}$

(30) $\frac{dP}{dt} = 0.2P - 10 = 0.2(P - 50)$

a) $\int \frac{dP}{P-50} = \int 0.2\,dt$

$\ln|P-50| = .2t + c$

$|P-50| = e^{.2t+c}$

$P - 50 = \pm Ce^{.2t}$

$$\boxed{P = \pm Ce^{.2t} + 50}$$

b) $(0,40)$ $\quad 40 = Ce^{.2t} + 50$

$-10 = C$

$$\boxed{P = -10e^{.2t} + 50}$$

$(0,50)$ $\quad 50 = Ce^{.2t} + 50$

$0 = Ce^{.2t}$

$C = 0$

$$\boxed{P = 0}$$

$(0,60)$ $\quad 60 = Ce^{.2t} + 50$

$10 = C$

$$\boxed{P = +10e^{.2t} + 50}$$

(31) $\frac{dy}{dt} = 100 - y = -1(y - 100)$

a) $\int \frac{dy}{y-100} = \int -1\,dt$

$\ln|y-100| = -t + c$

$y - 100 = \pm Ce^{-t}$

$$\boxed{y = \pm Ce^{-t} + 100}$$

c) $(0,25)$ $\quad \boxed{y = -75e^{-t} + 100}$

$(0,110)$ $\quad \boxed{y = +10e^{-t} + 100}$

(32) $\frac{dr}{dt} = \frac{k}{r}$ $(0,0)$

$\int r\,dr = \int k\,dt$

$\frac{1}{2}r^2 = kt + c \Big|_{(0,0)}$

$\therefore c = 0 \quad \therefore r^2 = 2kt$

$$\boxed{r = \pm\sqrt{2kt}}$$

$(16,400)$

$400^2 = 2k16$

$k = \frac{400^2}{32} = 5000$

$r = +\sqrt{10000t}$

$$\boxed{r = +100\sqrt{t}}$$