

**Review Unit 1: Functions, Graphs & Limits**

Prepare for TEST date \_\_\_\_\_

**DIRECTIONS: Complete all review questions on a separate paper in your notebook. Show all work.****Non-Calculator (do not use a calculator on these questions)**

1. Are the following functions continuous on the given intervals? Explain why or why not using the definition of continuity.

a.  $f(x) = \frac{1}{x-2}$  on  $[0,3]$     b.  $f(x) = \frac{1}{x-2}$  on  $[-1,1]$     c.  $f(x) = \frac{e^x}{e^x-1}$  on  $[-1,1]$

2. Use the function  $f(x)$  to find

$$f(x) = \begin{cases} x^2 - 1 & \text{for } -1 \leq x < 0 \\ 2x & \text{for } 0 < x < 1 \\ 1 & \text{for } x = 1 \\ -2x + 4 & \text{for } 1 < x < 2 \\ 0 & \text{for } 2 < x < 3 \end{cases}$$

a.  $\lim_{x \rightarrow a} f(x)$  for  $a = -1, 0, 1$

b. For what value(s) of  $x$  does the function have jump or removable discontinuities?

3. Discuss the continuity and asymptotes of  $y = \frac{x^2 + x - 2}{3x^2 - 4x + 1}$ .

4. Find  $k$  so that  $f(x)$  is continuous at  $x = 3$ :  $f(x) = \begin{cases} \frac{x^2 + 2x - 15}{x - 3} & \text{for } x \neq 3 \\ k & \text{for } x = 3 \end{cases}$

5. Let  $h(x) = \begin{cases} -2x + 3 & \text{for } x < -1 \\ x^2 + 3 & \text{for } x \geq -1 \end{cases}$

a. Find  $\lim_{x \rightarrow -1^-} h(x)$

b. Find  $\lim_{x \rightarrow -1^+} h(x)$

c. Find  $\lim_{x \rightarrow -1} h(x)$

d. Is the function continuous at  $x = -1$ ? Explain.

6. (1986 AB 4) Let  $f(x)$  be a function defined as follows  $f(x) = \begin{cases} |x-1| + 2 & \text{for } x < 1 \\ ax^2 + bx & \text{for } x \geq 1 \end{cases}$

a. If  $a = 2$  and  $b = 3$ , is  $f$  continuous for all  $x$ ? Justify your answer.

b. Describe all values of  $a$  and  $b$  for which  $f$  is a continuous function.

8. Evaluate the limits analytically using limit properties or algebraic strategies. Show all work.

a. $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 5}}{\sqrt[3]{50x - 3}}$	b. $\lim_{x \rightarrow 12} \frac{12 - x}{\frac{1}{x} - \frac{1}{12}}$	c. $\lim_{x \rightarrow 0} \frac{\sqrt{144 + x} - 12}{x}$	d. $\lim_{x \rightarrow 12} \frac{x^2 + 13x + 12}{x^2 + 10x - 24}$
e. $\lim_{x \rightarrow -\infty} \frac{4x - 7x^5 + 18}{5 + x - 35x^2}$	f. $\lim_{x \rightarrow \infty} \frac{-6x^5 + 2x - 11}{3x^5 + 4x^3 + 9x + 1}$	g. $\lim_{x \rightarrow \infty} \frac{1 + 3x^2 - 4x^3}{5x^7 - x^2 + x + 7}$	h. $\lim_{x \rightarrow -\infty} \frac{\sqrt{64x^2 + 5x}}{\sqrt[3]{125x^3 + 8}}$
i. $\lim_{x \rightarrow 2} \frac{\sin(x - 2)}{(2 - x)}$	j. $\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{(3x)}$	k. $\lim_{x \rightarrow 7} \frac{ x - 7 }{7 - x}$	l. $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 5}}{\sqrt[3]{50x - 3}}$

9. Use the Intermediate Value Theorem to determine whether or not the function has a zero on the given interval. Find all zeros on the interval exact to 3-decimal places using a graphing utility.

a.  $f(x) = x^3 - 0.276x^2 - 0.140157x$  on  $x \in [-1, 1]$       b.  $f(x) = \begin{cases} x^2 - 2, & x < 0 \\ 2 - x^2, & x \geq 0 \end{cases}$  on  $x \in [-1, 1]$

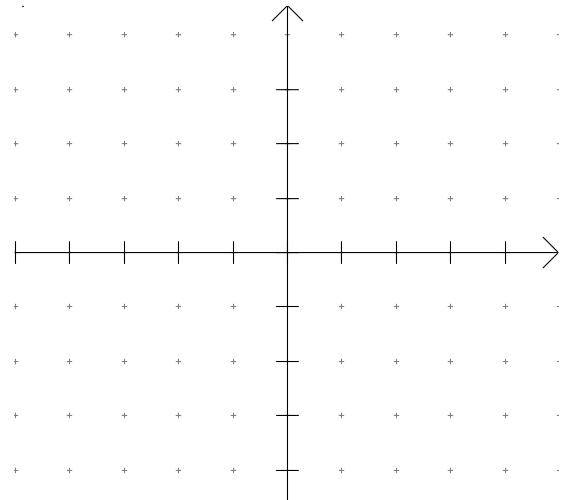
10. Sketch the graphs of the functions  $f(x) = 1 - x^2$  and  $h(x) = \cos(x)$  on the interval

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  satisfying the inequality  $f(x) \leq g(x) \leq h(x)$   
 $1 - x^2 \leq g(x) \leq \cos(x)$ . Use the graph to make a conclusion about  $\lim_{x \rightarrow 0} g(x)$  and explain how you arrived at your conclusion.

**Calculator (You may use a calculator on these questions)**

11. For the function  $f(x) = 3 + 0.6x + \frac{|x - 2|}{x - 2}$

- Find a complete graph of  $f(x)$ .
- Where is the function discontinuous?
- Evaluate  $f(1.9)$ ,  $f(1.99)$  and  $f(1.999)$ .
- What is  $\lim_{x \rightarrow 2^-} f(x)$ ?
- Evaluate  $f(2.1)$ ,  $f(2.01)$  and  $f(2.001)$ .
- What is  $\lim_{x \rightarrow 2^+} f(x)$ ?



12. Let  $f(x) = \frac{|x + 1|}{x - 2}$ .

- Find  $\lim_{x \rightarrow -\infty} f(x)$ .
- Find  $\lim_{x \rightarrow \infty} f(x)$ .
- Is this function continuous at  $x = 2$ ? Explain.

NOTE: In preparation for your test, you should review classwork, homework, quizzes and this review sheet. It is best to study for a test by DOING problems again and not just “looking them over.” DOING builds muscle memory so that you are able to answer questions on the test efficiently and accurately. You’ve got to know your stuff cold so you have enough time to complete the test during one class period.