<u>Fill in the Blanks</u> for the Big Topics in Chapter 5: The Definite Integral $\int_a^b f(t)dt$

- Estimating an integral using a Riemann sum:
  1. The <u>Left rule</u> uses the left endpoint of each subinterval.
  2. The <u>Right rule</u> uses the right endpoint of each subinterval.
  3. The <u>Midpoint rule</u> uses the midpoint of each subinterval.
  4. The <u>Trapezoid rule</u> uses the average from the left and right rules, i.e.

  Formula for Trapezoid estimate using Left and Right estimates: $Trap(n) =$

- If the graph of f is increasing on [a, b], then ______________ $\leq \int_a^b f(x)dx \leq$ ______________
- If the graph of f is decreasing on [a, b], then ______________ $\leq \int_a^b f(x)dx \leq$ ______________

- If the graph of f is concave up on [a, b], then ______________ $\leq \int_a^b f(x)dx \leq$ ______________
- If the graph of f is concave down on [a, b], then ______________ $\leq \int_a^b f(x)dx \leq$ ______________

- ____________________ $= \int_a^b F'(t)dt =$ total change of $F(t)$ between $t = a$ and $t = b$

- Average value of f from a to b = __________________________

- If f is even, then $\int_{-a}^{a} f(x)dx =$ ______________________________

- If g is odd, then $\int_{-a}^{a} g(x)dx =$ ______________________________

- Given $a \leq x \leq b$, $\int_b^a f(x)dx =$ ______________________________

- $\int_a^c f(x)dx + \int_c^b f(x)dx =$ ____________________________________

- $\int_a^b (f(x) + g(x))dx =$ ______________________________________

- $\int_a^b cf(x)dx =$ ____

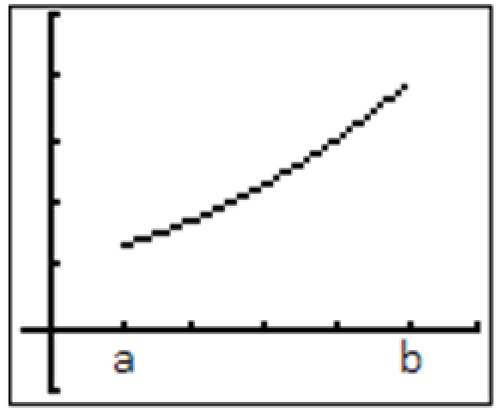
<u>Ch 5 Review: Free Response</u>

<u>Non-Calculator</u>

1. Calculate the exact value of $y = \int_0^3 \sqrt{9 - x^2}\,dx$. (Use the geometric formula.)

2. It is known that $\int_0^{12} f(x)dx = 18$ and $\int_0^9 f(x)dx = 10$, find $\int_9^{12} f(x)dx$.

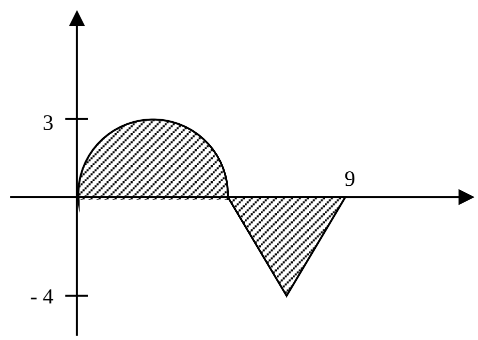
3. Show the following on the graph:
   a) $f(b) - f(a)$
   b) a line whose slope is $\dfrac{f(b) - f(a)}{b - a}$
   c) an area $F(b) - F(a)$ where $F' = f$
   d) $f(a)(b - a)$
   e) average value of $f = \dfrac{1}{b-a}\int_a^b f(x)dx$



<u>Calculator</u>

4. Set up an integral and calculate the average velocity of $v(t) = 50(1.2)^t$ in the first 10 seconds if t represents seconds and v represents ft/sec.

5. Find the area under the curve ---------------------→
   consisting of semi-circle and triangle



6. a) Graph $f(x) = x^3 - x^2 - 12x$.

   b) Find $\int_{-4}^{5} f(x)dx$.

   c) Set up integrals that represent the total area between the curve and the x-axis.

   d) Find the average value of the function on the interval [0, 3] (Always show your work and give the answer to the third decimal place.)

7. If $\int_0^5 f(x)dx = 5$ and $\int_0^2 f(x)dx = 3$, then $\int_2^5 f(x)dx$?

8. A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per sec).
   a) Find the displacement of the particle during the time period $1 \le t \le 4$.
   b) Find the distance traveled during this time period.

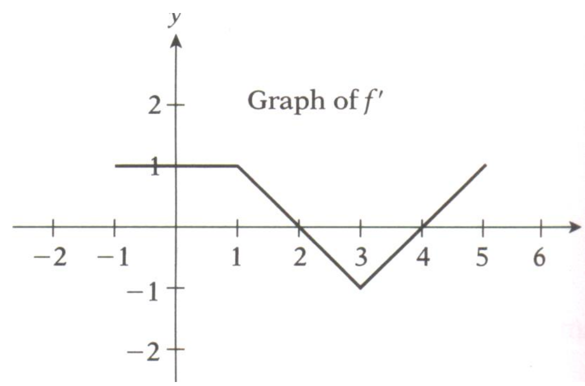
9. Use the Midpoint Rule with $n = 5$ to approximate $\int_1^2 \frac{1}{x}dx$.

Chapter 5 Review: Multiple Choice

Non-Calculator

1. $\int_0^2 |x-1|\,dx =$ (A) 0 (B) 1 (C) ½ (D) 2 (E) -1

For questions 2-5, $f(x)=\int_0^x f'(t)dt$ and the graph of *f'* is shown.



2. Which of the following is/are true?

   I. $f(-1) = -1$ II. $f(1) < f(3)$ III. $f'(1) < f'(3)$

   (A) I only (B) II only (C) III only
   (D) I and II (E) I, II and III

3. Which of the following is/are true about the graph of *f*?

   I. *f* is increasing on (-1, 2) only
   II. *f* is increasing on (-1, 2) and (4, 5)
   III. *f* is decreasing on (1, 3)

   (A) I only (B) II only (C) III only (D) I and III (E) none

4. Which of the following is/are true about the graph of *f*?

   I. *f* is concave up on (-1, 1)
   II. *f* is concave up on (1, 3)
   III. *f* is concave down on (3, 5)

   (A) I only (B) II only (C) III only (D) II and III (E) none

5. Which of the following is/are true about the graph of *f*?

   I. *f* has a relative minimum at $x = 2$
   II. *f* has a relative minimum at $x = 4$
   III. *f* has a relative maximum at $x = 2$

   (A) I only (B) II only (C) III only (D) I and II (E) II and III

6. The integral $\int_{-4}^{4}\sqrt{16-x^2}\,dx$ gives the area of

   (A) a circle of radius 4 (B) a semicircle of radius 4
   (C) a quarter of a circle of radius 4 (D) an ellipse whose minor axis is 4

7. If $f(x)$ is continuous on the interval $[a, b]$ and $a < c < b$, then $\int_c^b f(x)dx$ is equal to

   (A) $\int_a^c f(x)dx+\int_c^b f(x)dx$ (B) $\int_a^c f(x)dx-\int_a^b f(x)dx$
   (C) $\int_a^c f(x)dx+\int_b^a f(x)dx$ (D) $\int_a^b f(x)dx-\int_a^c f(x)dx$ (E) $\int_a^c f(x)dx+\int_c^b f(x)dx$

Calculator

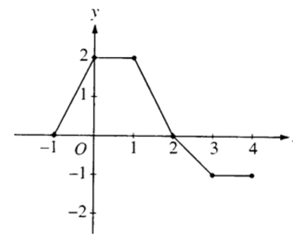
8. Let f be a continuous function on the closed interval $[0,2]$. If $2 \le f(x) \le 4$, then the greatest possible value of $\int_0^2 f(x)dx$ is

(A) 0 (B) 2 (C) 4 (D) 8 (E) 16

9. If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)dx = 7$, then $\int_1^3 f(x)\,dx =$

(A) –3 (B) 0 (C) 3 (D) 10 (E) 11

10. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown. What is the value of $\int_{-1}^{4} f(x)\ \ dx$?



(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

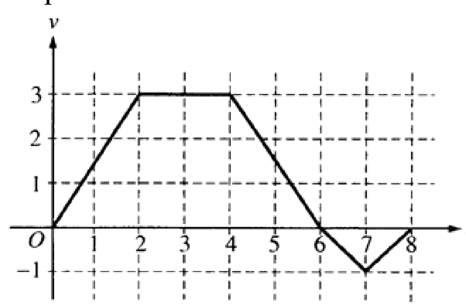
11. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx =$

(A) $F'(a) - F'(b)$ (B) $F'(b) - F'(a)$ (C) $F(a) - F(b)$
(D) $F(b) - F(a)$ (E) none of the above

12. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

Problems 13 and 14 refer to the diagram above. A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown.



13. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6
(D) 7 (E) 8

14. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11
(D) 8 (E) 6

15. If the definite integral $\int_0^2 e^{x^2} dx$ is first approximated by using two <u>inscribed</u> rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

(A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

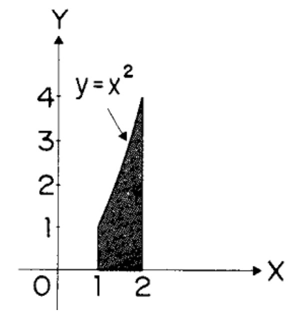
16. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

17. Find the average value of $y = \sqrt[3]{x+3}$ on the interval [-3, -2]

(A) 0.681 (B) 0.75 (C) 0.909 (D) 1.282 (E) 2.280

18. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.



(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

19. The function f is continuous on the closed interval $[2, 8]$ and has values that are given in the table. Using the subintervals $[2, 5]$, $[5, 7]$, and $[7, 8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

x	2	5	7	8
$f(x)$	10	30	40	20

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

20. During the worst 4-hour period of a hurricane the wind velocity, in miles per hour, is given by $v(t) = 5t - t^2 + 100,\ 0 \le t \le 4$. The average wind velocity during this period (in mph) is

(A) 10 (B) 100 (C) 102 (D) 104.667 (E) 108.667

21. If a factory continuously dumps pollutants into a river at the rate of $\frac{\sqrt{t}}{180}$ tons per day, then the amount dumped after 7 weeks is approximately

(A) 0.07 ton (B) 0.90 ton (C) 1.55 tons (D) 1.9 tons (E) 1.27 tons

Answers:

1. B	**2. A**	**3. B**	**4. E**	**5. E**	**6. B**	**7. D**
8. D	**9. E**	**10. B**	**11. D**	**12. D**	**13. C**	**14. B**
15. D	**16. A**	**17. B**	**18. D**	**19. C**	**20. D**	**21. E**