DAY 57 NOTES

§3.9 Linear Approximation and the Derivative – Student Notes

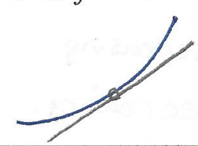
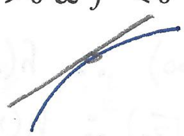
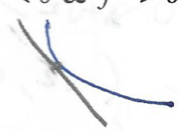
Tangent Line Approximations:
We can use the equation of the tangent line to approximate the value of a function at a particular value of x.

The concavity of the function tells us if an approximation made with the tangent line if an over-estimate (too high) or an under-estimate (too low.)

If a function is concave up, the tangent line will be BELOW the curve and any approximation made from the tangent line equation will be an underestimate.

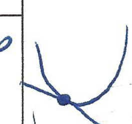
If a function is concave down, the tangent line will be ABOVE the curve and any approximation made from the tangent line equation will be an overestimate.

Sketch four portions of graphs satisfying the criteria given, then draw a point on each of the portions and draw a tangent line to the curve at that point. Do your pictures illustrate the conclusions you made above?

inc ccu	inc ccd	dec ccu	dec ccd.
$f'>0$ & $f''>0$	$f'>0$ & $f''<0$	$f'<0$ & $f''>0$	$f'<0$ & $f''<0$
			

For each question below, write the equation of the tangent line to the curve at the designated value of x. Use the tangent line equation to approximate the value of the function at the given x-value. Finally use the 2nd Derivative and concavity to justify whether the tangent line approximation is too high or too low.

Function & $x=a$	Tangent line equation at $x=a$	Tangent line approximation at $x=a$	Second Derivative evaluated at $x=a$	Is the tangent line approximation an overestimate or underestimate? Justify using f"
1 $f(x)=\sqrt{x}$ $x=49$ $f'(x)=\frac{1}{2\sqrt{x}}$ $f'(49)=\frac{1}{14}$ $f(49)=7$	$y=\frac{1}{14}(x-49)+7$	$f(50)\approx$ $\frac{1}{14}(1)+7=\frac{99}{14}$	$f''(49)=$ $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$ $f''(x)=\frac{-1}{4}x^{-\frac{3}{2}}$ $f''(x)=\frac{-1}{4x^{3/2}}$ $f''(49)=\frac{-1}{4(7)^3}<0$	f is concave down at $x=49$ ∴ tangent line approximation $f(50)=7\frac{1}{14}$ an overestimate
2 $f(x)=\frac{1}{x}$ $x=1$ $f'(x)=\frac{-1}{x^2}$ $f'(1)=-1$ $f(1)=1$	$y=-1(x-1)+1$	$f(1.1)\approx$ $=-1(.1)+1$ $=1-.1$ $=.99$	$f''(1)=$ $f'(x)=-x^{-2}$ $f''(x)=2x^{-3}$ $f''(x)=\frac{2}{x^3}\Big\vert_{x=1}$ $f''(1)=2>0$	f is concave up at $x=1$ ∴ tangent line approx $f(1.1)=.99$ is an underestimate.



HW. * $\boxed{\text{DAY 57}}$

		ESTIMATE TANGENT LINE APPROXIMATION ≈	$f''(a) =$	
3 $f(x)=\ln(x)$ $x=e$ $f'(x)=\frac{1}{x}$ $f'(e)=\frac{1}{e}$ $f(e)=1$	$y=\frac{1}{e}(x-e)+1$	$f(3)\approx$ $\frac{1}{e}(3-e)+1$ ≈ 1.103 ≈ 1.104	$f''(e)=$ $f'(x)=\frac{1}{x}$ $f''(x)=\frac{-1}{x^2}$ $f''(e)=\frac{-1}{e^2}<0$	f is ccd @ e ∴ $f(3)\approx 1.104$ is an overestimate.
4 $g(x)=\frac{1}{\sqrt{1+x}}$ $x=0$ $g'(x)=\frac{-1}{2(1+x)^{3/2}}$ $g'(0)=\frac{-1}{2}$ $g(0)=1$	$y=-\frac{1}{2}(x-0)+1$	$f(1.1)\approx$ $-\frac{1}{2}(.1)+1=$ $-\frac{1}{20}+1=$ $=\frac{19}{20}$	$f''(0)=$ $f'(x)=-\frac{1}{2}(x+1)^{-3/2}$ $f''(x)=-\frac{3}{4}(1+x)^{-5/2}$ $=\frac{-3}{4(1+x)^{5/2}}$ $f''(0)=\frac{-3}{4}<0$	f is ccd @ 0 ∴ $f(1.1)\approx\frac{19}{20}$ is an overestimate.
5 $h(x)=\frac{1}{1+x^2}$ $x=1$ $h'(x)=\frac{-1(2x)}{(1+x^2)^2}$ $h'(1)=\frac{-2}{4}=\frac{-1}{2}$ $h'(1)=\frac{1}{2}$	$y=-\frac{1}{2}(x-1)+\frac{1}{2}$	$f(1.01)\approx$ $\frac{-1}{2}(.01)+\frac{1}{2}$ $\frac{-1}{200}+\frac{1}{2}$ $\approx\frac{99}{200}$	$f''(1)=$ $h'(x)=\frac{(-2x)}{(1+x^2)^2}$ $h''(x)=\frac{(1+x^2)^2(-2)-(-2x)(2)(1+x^2)(2x)}{(1+x^2)^4}$ $=\frac{-2(1+x^2)\left[(1+x^2)-4x^2\right]}{(1+x^2)^4}=\frac{-2(1-3x^2)}{(1+x^2)^3}$ $f''(1)=\frac{+6}{1}>0$	f is ccu at 1.01 so $f(1.01)\approx\frac{99}{200}$ is underestimate.
6 $j(x)=\cos(x)$ $x=\frac{\pi}{6}$ $j'(x)=-\sin(x)$ $j'(\frac{\pi}{6})=\frac{1}{2}$ $j(\frac{\pi}{6})=\frac{\sqrt{3}}{2}$	$y=\frac{1}{2}(x-\frac{\pi}{6})+\frac{\sqrt{3}}{2}$	$f(0.5)\approx$ $\frac{1}{2}(\frac{1}{2}-\frac{\pi}{6})+\frac{\sqrt{3}}{2}$ ≈ 0.854	$f''(\frac{\pi}{6})=$ $f'(x)=-\sin x$ $f''(x)=-\cos x$ $f''(\frac{1}{2})=-\cos(\frac{\pi}{6})$ $f''(\frac{\pi}{6})=-\frac{\sqrt{3}}{2}<0$	f is ccd at $x=\frac{\pi}{6}$ so $f(\frac{1}{2})=0.854$ is overestimate.

SAVE UNTIL DAY 59

7 SHOW WORK IN YOUR NOTEBOOK: (NEED OPTIMIZATION TO ANSWER THIS QUESTION)

Let $h(x)$ be a function defined for all $x\neq 0$ such that $h(4)=-3$ and the derivative of h is given by

$$h'(x)=\frac{x^2-2}{x} \text{ for all } x\neq 0.$$

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, local minimum or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the tangent line to the graph of h at $x=4$.

(d) Does the line tangent to the graph of h at $x=4$ lie above or below the graph of h for $x>4$? Why?