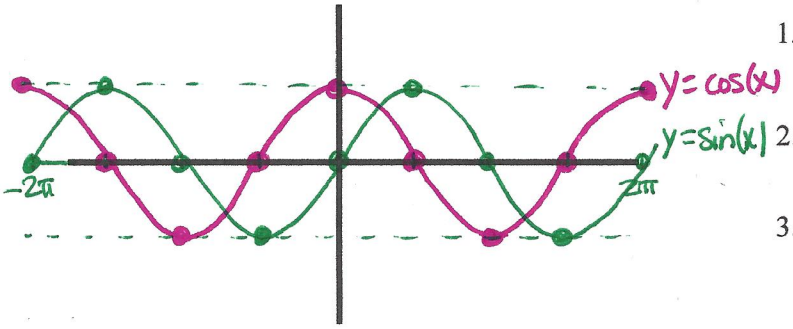
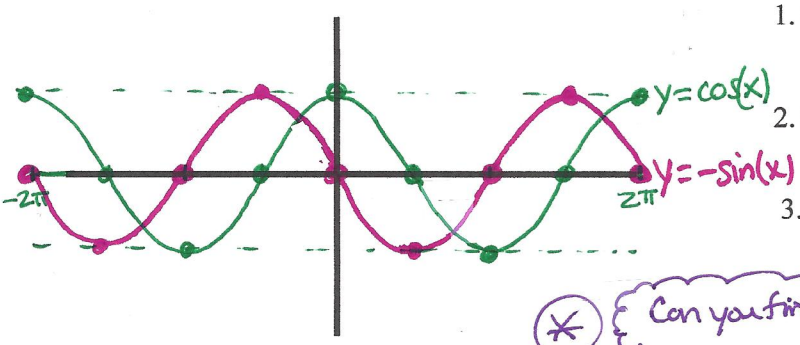
ANSWER KEY

§3.5 The Trigonometric Functions and Their Derivative -- Student Notes



1. Quickly sketch a labeled graph of $y = \sin x$ over $[-2\pi, 2\pi]$
2. Sketch a derivative of $y = \sin x$ in a different color.
3. What do you think is $\frac{d(\sin x)}{dx} = ?$ **cos(x)**



1. Quickly sketch a labeled graph of $y = \cos x$ over $[-2\pi, 2\pi]$
2. Sketch a derivative of $y = \cos x$ in a different color.
3. What do you think is $\frac{d(\cos x)}{dx} = ?$ **-Sin(x)**

(*) Can you find $\frac{d}{dx}(\cot x) = ?$ and $\frac{d}{dx}(\csc x) = ?$

4. Use the quotient rule to find $\frac{d(\tan x)}{dx} = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

$$\frac{d}{dx}(\tan x) = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

5. Use the quotient rule to find $\frac{d(\sec x)}{dx} = \frac{d}{dx}\left((\cos(x))^{-1}\right)$

$$\frac{d}{dx}(\sec(x)) = -1(\cos(x))^{-2}(-\sin(x))$$

$$= \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec(x) \cdot \tan(x)$$

Differentiate:

a) $y = 2\sin(3\theta)$

$\frac{dy}{d\theta} = 2\cos(3\theta) \cdot 3$

$\frac{dy}{d\theta} = 6\cos(3\theta)$

b) $y = \cos^2 x = (\cos(x))^2$

$\frac{dy}{dx} = 2\cos(x) \cdot (-\sin x)$

$= -2\sin(x)\cos(x)$

$\frac{dy}{dx} = -\sin(2x)$

ID: $2\sin(x)\cdot\cos(x) = \sin(2x)$

c) $y = \cos(x^2)$

$\frac{dy}{dx} = -\sin(x^2) \cdot 2x$

$\frac{dy}{dx} = -2x \cdot \sin(x^2)$

d) $y = e^{-\sin t}$

$\frac{dy}{dt} = \left(e^{-\sin t}\right)(-\cos t)$

$\frac{dy}{dt} = (-\cos t)\left(e^{-\sin t}\right)$

e) $y = 2\tan(3t)$

$\frac{dy}{dt} = 2\sec^2(3t) \cdot 3$

$\frac{dy}{dt} = 6\sec^2(3t)$

f) $y = \tan(1-\theta)$

$\frac{dy}{d\theta} = \sec^2(1-\theta) \cdot (-1)$

$\frac{dy}{d\theta} = -\sec^2(1-\theta)$

g) $y = \cos(x)\sin(x)$ product rule

$\frac{dy}{dx} = -\sin(x) \cdot \sin(x) + \cos(x) \cdot \cos(x)$

$= \cos^2 x - \sin^2 x$

$= \cos(2x)$

ID: $\cos(2x) = \cos^2 x - \sin^2 x$

h) $y = e^x \sin x$ product rule

$\frac{dy}{dx} = e^x \cdot \sin x + e^x \cdot \cos x$

$\frac{dy}{dx} = e^x(\sin x + \cos x)$

i) $y = \sin(3x) + \cos(2x)$

$\frac{dy}{dx} = \cos(3x) \cdot 3 - \sin(2x) \cdot 2$

$\frac{dy}{dx} = 3\cos(3x) - 2\sin(2x)$