① $g'(x) = 10x$ ② $g'(x) = -6x$ ③ $g'(x) = 5x^4$ ④ $g'(x) = 2x+1$ ⑤ $g'(x) = \frac{(1)(2t+1)-(t-7)(2)}{(2t+1)^2} = \frac{(2t+1)-2t+14}{(2t+1)^2} = \frac{15}{(2t+1)^2}$

⑥ $g'(x) = 2$ and $g'(x) = 3x^2-1$

DERIVATIVES of INNER FUNCTIONS

§3.4 The Chain Rule – Student Notes

The Chain Rule. If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(\boxed{g(x)})\, g'(x)$$

1. Find $F'(x)$ if $F(x) = \sqrt{5x^2+3} = (5x^2+3)^{\frac{1}{2}}$

$F'(x) = \frac{1}{2}(5x^2+3)^{-\frac{1}{2}} \cdot (10x)$

$F'(x) = \dfrac{10x}{2\sqrt{5x^2+3}}$

2. Differentiate $y = \dfrac{1}{(1-3x^2)^3} = (1-3x^2)^{-3}$

$\frac{dy}{dx} = -3(1-3x^2)^{-4} \cdot (-6x)$

$\frac{dy}{dx} = \dfrac{+18x}{(1-3x)^4}$

3. Differentiate $y = (x^5-1)^{1000}$

$\frac{dy}{dx} = 1000(x^5-1)^{999} \cdot (5x^4)$

$\frac{dy}{dx} = 5000x^4(x^5-1)^{999}$

4. Find $f'(x)$ if $f(x) = \dfrac{1}{\sqrt[3]{x^2+x+1}} = (x^2+x+1)^{-\frac{1}{3}}$

$f'(x) = -\frac{1}{3}(x^2+x+1)^{-\frac{4}{3}} \cdot (2x+1)$

$f'(x) = \dfrac{-(2x+1)}{3(x^2+x+1)^{4/3}}$

5. Differentiate $g(t) = \left(\dfrac{t-7}{2t+1}\right)^5$

$g'(t) = 5\left(\frac{t-7}{2t+1}\right)^4 \cdot \frac{15}{(2t+1)^2}$

$g'(t) = \dfrac{75}{(2t+1)^2} \cdot \left(\dfrac{t-7}{2t+1}\right)^4$

$g'(t) = \dfrac{(75)(t-7)^4}{(2t+1)^5}$

6. Differentiate $y = (2x+1)^5(x^3-x+1)^4$

$\frac{dy}{dx} = (2x+1)^5 \cdot 4(x^3-x+1)^3 \cdot (3x^2-1) + (x^3-x+1)^4 \cdot 5(2x+1)^4 \cdot (2)$

$\frac{dy}{dx} = (2x+1)^4 \cdot (x^3-x+1)^3 \left[(2x+1)(4)(3x^2-1) + (x^3-x+1)\cdot 10\right]$

OK → $\frac{dy}{dx} = (2x+1)^4(x^3-x+1)^3 \underbrace{\left[4(2x+1)(3x^2-1) + 10(x^3-x+1)\right]}$

$\frac{dy}{dx} = (2x+1)^4(x^3-x+1)^3\left[24x^3+12x^2-8x-4+10x^3-10x+10\right]$

$\frac{dy}{dx} = (2x+1)^4(x^3-x+1)^3(34x^3+12x^2-18x+6)$

§3.4 CHAIN RULE

$(c, f(c)) = \text{POINT}$
$f'(c) = \text{slope of tangent}$ → TANGENT LINE EQUATION: $y = f'(c)(x-c) + f(c)$

For #7-10: Write the equation of the tangent line at the x-value where you evaluated the derivative. Is this tangent line above or below the curve of the function? How do you know?
↳ check $f''(x) > 0$ or $f''(x) < 0$

7. Differentiate $y = e^{(x^2)}$ and evaluate $f'(-4)$

$$\frac{dy}{dx} = e^{x^2} \cdot (2x)$$

$$\left.\frac{dy}{dx}\right|_{x=-4} = -8e^{16}$$

$$(-4, f(-4)) = (-4, e^{16})$$

TANGENT LINE:

$$\boxed{y = -8e^{16}(x+4) + e^{16}}$$

$$\frac{d^2y}{dx^2} = (e^{x^2} \cdot 2x) \cdot 2x + e^{x^2}(2)$$

$$\frac{d^2y}{dx^2} = 2e^{x^2}(2x^2+1)$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=-4} = 2e^{16}(33) > 0$$

∴ f(x) is concave up at $x = -4$ & tangent line is below the curve f(x)

8. If $f(x) = 3^{2x-1}$, then $f'(2) = ?$

$$f'(x) = (\ln 3)(3^{2x-1}) \cdot (2)$$

$$f'(x) = (\ln 9)\, 3^{2x-1}$$

$$f'(2) = (\ln 9)(27)$$

$$(2, f(2)) = (2, 27)$$

TANGENT LINE:

$$\boxed{y = (\ln 9)(27)(x-2) + 27}$$

$$f''(x) =$$

$$f'' = (\ln 9)(\ln 3)\, 3^{2x-1} \cdot 2$$

$$f'' = (\ln 9)^2 \cdot 3^{2x-1}$$

$$f''(2) = (\ln 9)^2 \cdot 27 > 0$$

∴ f(x) is concave up at $x = 2$ and tangent line is below the curve f(x).

9. If $y = e^{(2x^3-3x+4)}$, then $y'(-1) = ?$

$$\frac{dy}{dx} = \left(e^{2x^3-3x+4}\right)(6x^2-3)$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = \left(e^{-2+3+4}\right)(3) = 3e^5$$

$$(-1, f(-1)) = (-1, e^5)$$

TANGENT LINE

$$\boxed{y = 3e^5(x+1) + e^5}$$

$$\frac{d^2y}{dx^2} = \left(e^{2x^3-3x+4}\right)(6x^2-3) \cdot (6x^2-3) + \left(e^{2x^3-3x+4}\right)(12x)$$

$$\frac{d^2y}{dx^2} = \left(e^{2x^3-3x+4}\right)\left[(6x^2-3)^2 + 12x\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=-1} = e^5\left[9-12\right] = -3e^5 < 0$$

∴ f(x) is concave down at $x = -1$ and the tangent line is above the curve f(x).

10. If $f(x) = 5^{2x}$, then $f'(3) = ?$

$$f'(x) = (\ln 5)\, 5^{2x} \cdot 2 = (\ln 25)\, 5^{2x}$$

$$f'(3) = (\ln 25) \cdot 5^6$$

$$(3, f(3)) = (3, 5^6)$$

TANGENT LINE: $y = (\ln 25) 5^6 (x-3) + 5^6$

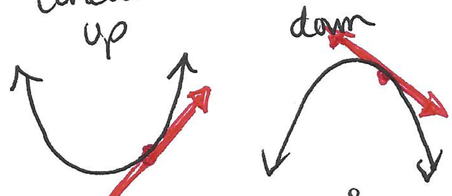
$$f''(x) = (\ln 25)^2\, 5^{2x}$$

$$f''(3) = (\ln 25)^2 \cdot 5^6 > 0$$

∴ f(x) is concave up at $x = 3$ and the tangent line is below the curve of f(x)

Concave up

Concave down



tangent line below f(x)

tangent line above f(x)