§3.3 KEY

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

§3.3 Product and Quotient Rules -- Student Notes

The Product Rule If f and g are both differentiable, then:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Example 1: $h(x) = (x^3+1)(2^x)$

$$h'(x) = (3x^2)(2^x) + (x^3+1)(\ln 2 \cdot 2^x)$$

$$h'(x) = (2^x)[3x^2 + (x^3+1)\cdot \ln 2]$$

1. If $h(x) = xe^x$, find $h'(x)$.

$$h'(x) = (1)(e^x) + (x)(e^x)$$

$$h'(x) = e^x + xe^x$$

$$h'(x) = e^x(1+x)$$

$\sqrt{t} = t^{\frac{1}{2}}$

2. Use two different methods to differentiate the function: $h(t) = \sqrt{t}(1-t)$.

$$h(t) = \sqrt{t}(1-t)$$

$$h(t) = \sqrt{t} - t^{3/2}$$

$$h'(t) = \frac{1}{2}t^{-\frac{1}{2}} - \frac{3}{2}t^{\frac{1}{2}}$$

$$h'(t) = \underbrace{\frac{1}{2\sqrt{t}} - \frac{3\sqrt{t}}{2}}_{LCD}$$

$$h'(t) = \frac{1-3t}{2\sqrt{t}}$$

$$h'(x) = \left(\frac{1}{2}t^{-\frac{1}{2}}\right)(1-t) + (\sqrt{t})(-1)$$

$$= \underbrace{\frac{1-t}{2\sqrt{t}} - \sqrt{t}}_{LCD} = \frac{1-t-2t}{2\sqrt{t}} = \frac{1-3t}{2\sqrt{t}}$$

The Quotient Rule If f and g are both differentiable, then:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Example 2: $h(x) = \dfrac{x^2+1}{x^3-5}$

$$h'(x) = \frac{(2x)(x^3-5) - (x^2+1)(3x^2)}{(x^3-5)^2}$$

$$= \frac{(2x^4-10x) - (3x^4+3x^2)}{(x^3-5)^2}$$

$$= \frac{-x^4+3x^2-10x}{(x^3-5)^2}$$

$$= \frac{-x(x^3-3x+10)}{(x^3-5)^2}$$

3. Let $y = \dfrac{x^2+x-2}{x^3+6}$, then find $\dfrac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{(x^3+6)(2x+1) - (x^2+x-2)(3x^2)}{(x^3+6)^2}$$

$$\frac{dy}{dx} = \frac{(2x^4+x^3+12x+6) - (3x^4+3x^3-6x^2)}{(x^3+6)^2}$$

$$\frac{dy}{dx} = \frac{-x^4+4x^3-6x^2+12x+6}{(x^3+6)^2}$$

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$: $\frac{lo \cdot D(hi) - hi \cdot D(lo)}{(lo)^2}$ $\frac{up \cdot D(down) - down \cdot D(up)}{(down)^2}$

4. Find an equation of the tangent line to the curve $y = \frac{e^x}{x}$ at the point $(1, e)$.

$$\frac{dy}{dx} = \frac{(x)(e^x) - (e^x)(1)}{x^2}$$

$$\frac{dy}{dx} = \left.\frac{e^x(x-1)}{x^2}\right|_{x=1} = 0$$

Since the slope of f(x) at x=1 is 0 the tangent line to f(x) at (1,e) is horizontal.

TANGENT Line : $y = e$

5. Given $f(x) = \frac{3x^2 + 2\sqrt{x}}{x}$, find $\frac{d}{dx}[f(x)]$.

Hint: re-write f(x) before taking the derivative.

$$f(x) = 3x + 2x^{-1/2}$$

$$f'(x) = 3 + -1x^{-3/2}$$

$$f'(x) = 3 + \frac{-1}{x^{3/2}} = \frac{3x^{3/2} - 1}{x^{3/2}}$$

$$f'(x) = \frac{3x^{3/2} - 1}{x^{3/2}}$$

Practice. Find each of the following derivatives:

6. $\frac{d}{dx}\left[(x^3 - 2x + 1)(x^4 + x - 3)\right]$

$$= (3x^2 - 2)(x^4 + x - 3) + (x^3 - 2x + 1)(4x^3 + 1)$$

7. $\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) = \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$

$$= \frac{(2x)\left[x^2 - 1 - x^2 - 1\right]}{(x^2-1)^2}$$

$$= \frac{2x(-2)}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

8. $\frac{d}{dx}\left(\frac{2x+1}{3^x}\right)$

$$= \frac{(3^x)(2) - (2x+1)(\ln 3 \cdot 3^x)}{(3^x)^2}$$

$$= \frac{\cancel{3^x}\left[2 - (\ln 3)(2x+1)\right]}{\cancel{3^x}\, 3^x}$$

9. $\frac{d}{dx}(2^x \cdot e^x)$

$$= (\ln 2 \cdot 2^x)(e^x) + (2^x)(e^x)$$

$$= 2^x e^x (\ln 2 + 1)$$

10. $\frac{d}{dx}(\sqrt{x}\, e^x)$

$$= \frac{1}{2\sqrt{x}} \cdot e^x + \sqrt{x} \cdot e^x$$

$$= e^x \left(\frac{1}{2\sqrt{x}} + \frac{\sqrt{x}}{1}\right)$$

$$= e^x \left(\frac{1 + 2x}{2\sqrt{x}}\right)$$

$$= \frac{(e^x)(1 + 2x)}{2\sqrt{x}}$$

11. $\frac{d}{dx}\left(\frac{e^x}{1-5x}\right)$

$$= \frac{(1-5x)(e^x) - (e^x)(-5)}{(1-5x)^2}$$

$$= \frac{e^x(1 - 5x + 5)}{(1-5x)^2} = \frac{e^x(6-5x)}{(1-5x)^2}$$