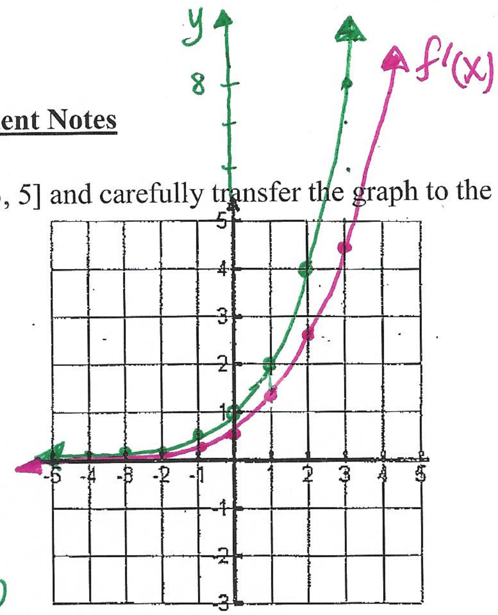
§3.2 KEY

§3.2 The Exponential Function – Student Notes



1. Use your calculator to graph $\boxed{f(x) = 2^x}$ with a window of [-5, 5] x [-3, 5] and carefully transfer the graph to the grid at the right.

   a. Find the domain of $f(x)$. $x \in \mathbb{R}$

   b. Find the range of $f(x)$. $y > 0$

   c. Where is $f(x)$ increasing? $x \in \mathbb{R}$

   d. Where is $f(x)$ decreasing? $\emptyset$ never

   e. Describe the concavity of $f(x)$. f(x) is concave up

   f. On the same axes sketch $\boxed{f'(x) \text{ using a different color.}}$ You can use your calculator by typing Y2=nderiv (Y1,X, X). *(nderiv is under "MATH"-"8" Y1 is under "VARS" "Y-VARS"-"FUNC")*

   Left of $\boxed{=}$ change to $\boxed{-0}$ "Ball & Chain"

   g. Find the domain of $f'(x)$: $x \in \mathbb{R}$ range: $y > 0$

   h. Where is $f'(x)$ increasing? $x \in \mathbb{R}$ decreasing? $\emptyset$ never

   i. Describe the concavity of $f'(x)$. f'(x) is concave up

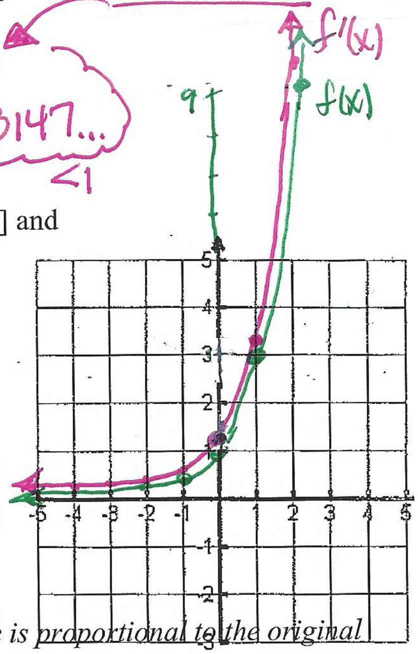
   j. What are the *y*-intercepts of each? $y = 2^x$ (0,1) $f'(x)$ (0, 0.69315)

   k. How do the two graphs differ? f'(x) is a scale factor shorter than f(x)

   l. Estimate, to the best of your ability, the equation of $f'(x)$. (Try various numerical values until your graph of $f(x)$ matches the graph of $f'(x)$.)

   $f'(x) = (\ln 2)(2^x)$

   ln2 = 0.693147... <1



2. Use your calculator to graph $\boxed{f(x) = 3^x}$ with a window of [-5, 5] x [-3, 5] and carefully transfer the graph to the grid at the right.
   On the same axes sketch $\boxed{f'(x) \text{ using a different color.}}$
   Can use the calculator again to find $f'(x)$

   Estimate, to the best of your ability, the equation of $f'(x)$.

   $f'(x) = (\ln 3)(3^x)$

   ln 3 = 3.295837... >1

3. You should have discovered that for exponential functions, the *derivative is proportional to the original function; i.e.,* $f'(x) = kf(x)$. Note that the constant of proportionality for $f(x) = 2^x$ is less than one and that the

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constant of proportionality for $f(x)=3^x$ is greater than one. Therefore, if $f(x)=a^x$, then for some value of a between 2 and 3, the constant of proportionality equals one. That means if $f(x)=a^x$, then for some a, $f(x)=f'(x)$. What value of a has the property that $f(x)=f'(x)$?

$f(x)=$ e^x $\Longrightarrow$ $f'(x)=e^x$ verify on TI-84

But, in general, if $f(x)=a^x$, then $f'(x)=$ $(\ln a)(a^x)$

Exponential Practice Find $f'(x)$ for each of the following functions.

1. $f(x)=4^x$
$f'(x)=(\ln 4)\,4^x$

2. $f(x)=e^x$
$f'(x)=e^x$

3. $f(x)=6^x$
$f'(x)=(\ln 6)\,6^x$

4. $f(x)=8^x$
$f'(x)=(\ln 8)\,8^x$

5. $f(x)=x^4$
$f'(x)=4x^3$

6. $f(x)=\pi^x$
$f'(x)=(\ln\pi)\,\pi^x$

7. $f(x)=7^x$
$f'(x)=(\ln 7)\,7^x$

8. $f(x)=9^x$
$f'(x)=(\ln 9)\,9^x$

9. $f(x)=x^e$
$f'(x)=ex^{e-1}$

10. $f(x)=2\cdot 3^x$
$f'(x)=2(\ln 3)\,3^x$
$f'(x)=(\ln 9)\,3^x$

11. $f(x)=2^x+x^2$
$f'(x)=(\ln 2)2^x+2x^1$

12. $f(x)=4^x-3^x$
$f'(x)=(\ln 4)4^x-(\ln 3)3^x$

13. $f(x)=e^x+2x^3$
$f'(x)=e^x+6x^2$

14. $f(x)=2^{x+3}$
$f(x)=2^3\cdot 2^x$
$f'(x)=2^3(\ln 2)\,2^x$
$f'(x)=(\ln 2)\,2^{x+3}$

15. $f(x)=e^{x+\pi}$
$f(x)=(e^\pi)(e^x)$
$f'(x)=e^\pi\cdot e^x$
$f'(x)=e^{x+\pi}$

★ 16. $f(x)=2^{3x+5}$
$f(x)=2^5\cdot 8^x$
$f'(x)=2^5\cdot(\ln 8)\,8^x=2^5(\ln 8)\,2^{3x}=(\ln 8)\,2^{3x+5}$