Key §3.1

§3.1 Derivatives of Powers and Polynomials -- Student Notes

The Power Rule If n is any real number, then: $\boxed{\frac{d}{dx}\left(x^n\right) = nx^{n-1}}$ ← POWER RULE

① Rewrite f(x) & LABEL f(x)
② Use Power Rule to take derivative label f'(x)

1. Differentiate: a) $f(x) = \frac{1}{x^2}$

$f(x) = x^{-2}$
$f'(x) = -2x^{-3}$
$f'(x) = \frac{-2}{x^3}$

b) $y = \sqrt[5]{x^3}$

$f(x) = x^{3/5}$
$f'(x) = \frac{3}{5}x^{-2/5}$
$f'(x) = \frac{3}{5x^{2/5}}$

Let's see how TI-84 does this.

2. Find an equation of the tangent line to the curve $y = x\sqrt{x}$ at the point $(1, 1)$. Illustrate by graphing the curve and its tangent line.

$f(x) = x^{3/2}$ $f(1) = 1$
$f'(x) = \frac{3}{2}x^{1/2} \longrightarrow f'(1) = \frac{3}{2}$
$f'(x) = \frac{3}{2}\sqrt{x}$

TANGENT LINE TO CURVE
$y = \frac{3}{2}(x-1)+1$

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}f(x)$$

COEFFICIENTS GET TO HANG OUT IN FRONT.

The Sum/Difference Rule If f and g are both differentiable, then:

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$$

3. $\frac{d}{dx}\left(\underbrace{x^8 - 8x^5 + 2x^4 + 10x^2 - 6x + 5}_{f(x)}\right) = 8x^7 - 40x^4 + 8x^3 + 20x - 6 = f'(x)$

4. Find the points on the curve $y = x^4 - 10x^2 + 2$ where the tangent line is horizontal. → $f'(x) = 0$

$f'(x) = 4x^3 - 20x$
$f'(x) = 0 = 4x(x^2 - 5)$
$0 = 4x(x-\sqrt{5})(x+\sqrt{5})$
$x = 0, \sqrt{5}, -\sqrt{5}$

So f(x) has horizontal tangent lines to its curve @ $x = 0, \sqrt{5}, -\sqrt{5}$

§3.1 f(x) in green f'(x) in Pink {POSITIVE EXPONENTS}

Section 3.1 Practice A. Find the derivatives of each function. For #8-16 first re-write f(x) as the sum or difference of x-expressions raised to a real power. Do not leave negative exponents in your final answer.

1. $y = 0$

$f'(x) = 0$

2. $y = -4x + \pi$

$f'(x) = -4$

3. $y = 1.2x^2 - ex$

$f'(x) = 2.4x - e$

4. $y = \frac{2}{3}x$

$f'(x) = \frac{2}{3}$

5. $y = 5x^2 - 4x + 9\pi^3$

$f'(x) = 10x - 4$

6. $y = x^\pi + x^e + e^\pi$

$f'(x) = \pi x^{\pi-1} + e x^{e-1}$

6. $y = 2x^{\frac{1}{4}}$

$f'(x) = \frac{1}{2}x^{-3/4}$

$f'(x) = \frac{1}{2x^{3/4}}$

7. $y = \sqrt{x}$

$f(x) = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(x) = \frac{1}{2\sqrt{x}}$

8-16: Re-write f(x) before differentiating!

8. $y = \sqrt[3]{x} + \sqrt[3]{x^2} + \sqrt[3]{x^4}$

$f(x) = x^{1/3} + x^{2/3} + x^{4/3}$

$f'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-1/3} + \frac{4}{3}x^{1/3}$

$f'(x) = \frac{1}{3x^{2/3}} + \frac{2}{3x^{1/3}} + \frac{4x^{1/3}}{3}$

9. $y = \sqrt[4]{x} + \sqrt[4]{x^3} + \sqrt[4]{x^5}$

$f(x) = x^{1/4} + x^{3/4} + x^{5/4}$

$f'(x) = \frac{1}{4}x^{-3/4} + \frac{3}{4}x^{-1/4} + \frac{5}{4}x^{1/4}$

$f'(x) = \frac{1}{4x^{3/4}} + \frac{3}{4x^{1/4}} + \frac{5x^{1/4}}{4}$

10. $y = \frac{1}{x}$

$f(x) = x^{-1}$

$f'(x) = -1x^{-2}$

$f'(x) = \frac{-1}{x^2}$

11. $y = \frac{1}{x^2} + \frac{1}{x^3}$

$f(x) = x^{-2} + x^{-3}$

$f'(x) = -2x^{-3} + -3x^{-4}$

$f'(x) = \frac{-2}{x^3} + \frac{-3}{x^4}$

12. $y = \frac{1}{\sqrt{x}}$

$f(x) = x^{-1/2}$

$f'(x) = -\frac{1}{2}x^{-3/2}$

$f'(x) = \frac{-1}{2x^{3/2}}$

13. $y = \frac{1}{\sqrt[3]{x}}$

$f(x) = x^{-\frac{1}{3}}$

$f'(x) = -\frac{1}{3}x^{-4/3}$

$f'(x) = \frac{-1}{3x^{4/3}}$

14. $y = 4t^2 - \frac{5}{\sqrt{t}} + \frac{1}{t^3}$

$f(t) = 4t^2 - 5t^{-1/2} + t^{-3}$

$f'(t) = 8t + \frac{5}{2}t^{-3/2} - 3t^{-4}$

$f'(t) = 8t + \frac{5}{2t^{3/2}} - \frac{3}{t^4}$

15. $y = \frac{5t^4 - 3t^3 - 8t^2 + t}{t^3}$

$f(t) = 5t - 3 - 8t^{-1} + t^{-2}$

$f'(t) = 5 + 8t^{-2} - 2t^{-3}$

$f'(t) = 5 + \frac{8}{t^2} - \frac{2}{t^3}$

16. $y = \sqrt{x}(4x^3 - 5x^2 + 7)$

$f(x) = 4x^{7/2} - 5x^{5/2} + 7x^{1/2}$

$f'(x) = 14x^{5/2} + \frac{25}{2}x^{3/2} + \frac{7}{2}x^{-\frac{1}{2}}$

$f'(x) = 14x^{5/2} + \frac{25}{2}x^{3/2} + \frac{7}{2\sqrt{x}}$

17. Find $\frac{dy}{dx}$ if $y = \frac{x^3}{a} + bx^2 - cx$

$\frac{dy}{dx} = \frac{3}{a}x^2 + 2bx - c$

18. Find $\frac{dw}{dq}$ if $w = 3ab^2q^3$

coefficient

$\frac{dw}{dq} = 3ab^2(3q^2)$

$\frac{dw}{dq} = 9ab^2q^2$