§3.1 - §3.5 Applying the Derivative Rules using Tables

The purpose of this worksheet is to abstract the concept of the derivative rules by causing you to apply them to functions that you do not know. Two functions, $f(x)$ and $g(x)$, have the values and first derivatives shown in the table. Use this information to find the following.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-4	2	-2	-1	1
-3	1	-1	-2	2
-2	-2	1	0	3
-1	-1	4	2	1
0	0	5	1	0
1	2	3	0	-2
2	3	2	1	-1
3	3	1	-1	-3
4	1	-1	-2	-4

1. $h(x) = f(x) - g(x)$
   Find $h'(2)$

2. $h(x) = f(x) + 3g(x)$
   Find $h'(0)$

3. $h(x) = 2f(x) - 4g(x)$
   Find $h'(-3)$

4. $h(x) = 2f(x) - 1$
   Find $h'(3)$

5. $h(x) = 3g(x) - x^2$
   Find $h'(1)$

6. $h(x) = x\ f(x)$
   Find $h'(-1)$

7. $h(x) = [\ f(x)]^2$
   Find $h'(-3)$

8. $h(x) = f(x)g(x)$
   Find $h'(2)$

9. $h(x) = x^2 f(x)g(x)$
   Find $h'(-1)$

10. $h(x) = f(x)/g(x)$
    Find $h'(-2)$

11. $h(x) = f(3x)$
    Find $h'(-1)$

12. $h(x) = g\left(x^2\right)$
    Find $h'(-2)$

13. $h(x) = f\left(x^3 - x\right)$
    Find $h'(1)$

14. $h(x) = f(g(x))$
    Find $h'(4)$

15. $h(x) = g(f(x))$
    Find $h'(-3)$

16. $h(x) = [\ f(x)]^3\ g(-2x)$
    Find $h'(2)$

17. $h(x) = x^2/f(x)$
    Find $h'(-1)$

18. $h(x) = f(\ln\ x)/g(2x+1)$
    Find $h'(1)$

	ON YOUR PAPER	***ON CALCULATOR***
1 Given the function $f(x)$, **show the calculation** necessary to find the: a) average rate of change, average velocity, or slope of the secant on the interval $x \in [4.3, 5.6]$ ....>		
b) instantaneous rate of change, instantaneous velocity, or slope of the tangent line at $x = 4.95$ ....>		

2 The height of a projectile propelled from a platform 120 feet in the air with an initial velocity of 96 ft/sec is given by the function $h(t) = -\frac{1}{2}a_0 t^2 + v_0 t + h_0$. Note: Earth's gravitational constant is 32 ft/sec^2.

Write the equation for $h(t)$ =______________________________ and **show the calculation** necessary to find the:

a) average rate of change, average velocity, or slope of the secant on each of the intervals

Interval	$t \in [0,1]$	$t \in [1,2]$	$t \in [2,3]$	$t \in [3.012, 5.789]$	$t \in [4.218, 6.357]$
Algebraic Expression in terms of h(t)					
Evaluation (3-decimal accuracy)					

Since function is defined, write the expression for the slope of secant and evaluate the expression on the calculator. Record 3-decimal accuracy.

b) instantaneous rate of change, instantaneous velocity, or slope of the tangent line at $t = 3.724$ seconds..........>

c) Examine the values for the first three intervals what do they tell you about the behavior of the function. You should be able to conclude two specific ideas.

3 Given the table of values

$x_{(sec)}$	0	1	2	3	4	5	6	7
$f(x)_{(meters)}$	120	200	248	264	248	200	120	8

show the calculation necessary to find the:

ON PAPER: must pull values from the table and use in calculation

a) average rate of change, average velocity, or slope of the secant on the interval $x \in [4, 6]$...............>

b) instantaneous rate of change, instantaneous velocity, or slope of the tangent line at $x = 5$..............................>

Using appropriate **MATHEMATICAL NOTATION** to write what is required to justify Continuity & Differentiability.

4 Definition of Continuity in 3 parts.

a)

b)

c)

5 Definition of Differentibility.

§3.7 Implicit Differentiation -- A Brief Introduction -- Student Notes

Find these derivatives of these functions:

$y = \tan(x)$	$y = \sin(x)$	$y = e^x$
$\frac{d}{dx}(\tan(x)) =$	$\frac{d}{dx}(\sin(x)) =$	$\frac{d}{dx}(e^x) =$

Write the inverses of these functions:

$y = \tan(x)$	$y = \sin(x)$	$y = e^x$

How would we find the derivatives of these inverse functions?

$\frac{d}{dx}(\underline{\hspace{4cm}})$	$\frac{d}{dx}(\underline{\hspace{4cm}})$	$\frac{d}{dx}(\underline{\hspace{4cm}})$

Let's look at a brief introduction to <u>Implicit Differentiation</u> so that we can find the derivatives of these three inverse functions.

Up to now, we have worked explicitly, solving an equation for one variable *y* in terms of another variable *x*. For example, if you were asked to find $\frac{dy}{dx}$ for $2x^2 + y^2 = 4$, you would solve for *y* and get $y = \pm\sqrt{4 - 2x^2}$ and then take the derivative. This derivative requires the use of the chain rule.

Sometimes it is inconvenient, difficult or impossible to solve for *y*. In this case, we use <u>implicit differentiation</u>. It is imperative to note that anytime you see a *y*-variable you must think of *y* as a function of *x* just as in the notation: $y = f(x)$. Since I do not know the explicit form of *f(x)* I will apply the chain rule to indicate it's derivative.

Differentiating with respect to *x*:

$$\frac{d}{dx}\left[x^3\right] = 3x^2$$

Variables agree $\Rightarrow$ use power rule

variables agree

$$\frac{d}{dx}\left[y^3\right] = 3y^2\frac{dy}{dx}$$

Variables disagree $\Rightarrow$ use power rule and chain rule

variables disagree

$$\frac{d}{dx}\left[2x^5 + 3y\right] = 10x + 3\frac{dy}{dx}$$

variables disagree

variables agree

Practice: 1. $-3y^2 = x^4 + 5$ 2. $y^2 - 7y = \cos\left(x^3\right)$

Find $\dfrac{dy}{dx}$

Let's return to our 3 inverse functions and use implicit differentiation to find their derivatives:

$\dfrac{d}{dx}\left(\arctan\left(x\right)\right)$ $\dfrac{d}{dx}\left(\arcsin\left(x\right)\right)$ $\dfrac{d}{dx}\left(\ln\left(x\right)\right)$

Derivatives of some important Inverse functions (MEMORIZE THESE).

$\dfrac{d(\arctan x)}{dx} =$	$\dfrac{d(\arcsin x)}{dx} =$	$\dfrac{d(\ln x)}{dx} =$

Note: $\arctan x = \tan^{-1} x$ & $\arctan(\tan x) = \tan(\arctan x) = x$

$\arcsin x = \sin^{-1} x$ & $\arcsin(\sin x) = \sin(\arcsin x) = x$

Practice: Examples using the derivative rules we just found and applying rules we already learned:

a) $\dfrac{d(\arctan(t^2))}{dt}$ b) $\dfrac{d(\arcsin(\tan(\theta)))}{d\theta}$ c) $\dfrac{d\ln(x^2+1)}{dx}$

d) $\dfrac{d(t^2\ln t)}{dt}$ 3) $\dfrac{d(\sqrt{1+\ln(2y)})}{dy}$ f) $\dfrac{d(\cos(\sin^{-1} x))}{dx}$