AP Calculus AB— Unit 6 Motion with Integrals:
Position – Velocity – Acceleration – Speed – Total Distance – Displacement

ALWAYS STATE THESE

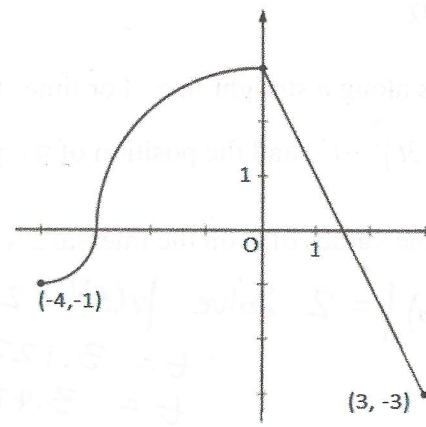
$\boxed{f = g' \quad f' = g''}$

LT: I can read and interpret an integral defined function.

number line analysis:

$f = g'$: -4 (−) -3 (+) 1.5 (−) 3 ; $-4 \searrow -3 \nearrow 1.5 \searrow 3$

$f' = g''$: -4 (+) -3 (+) 0 (−) 3 ; $-4 \cup -3 \cup 0 \frown 3$



Graph of f

5. Let f be the continuous function defined on [-4, 3]. The graph of f consists of two quarter circles and one line segment as shown in the figure above. Let g be the function given by $g(x) = \int_0^x f(t)dt$.

(a) Find the values of $g(-3)$, $g(1)$ and $g(3)$.

$g(-3) = \int_0^{-3} f(t)\,dt = -\frac{9\pi}{4}$ $g(1) = \int_0^1 f(t)\,dt = \frac{9}{4}$ $g(3) = \int_0^3 f(t)\,dt = 0$

(b) On what interval(s) is $g(x)$ decreasing and concave up?

$g(x)$ is decreasing on $(-4,-3)$ $(\frac{3}{2},3)$ b/c $f = g' < 0$
$g(x)$ is concave up on $(-4,0)$ b/c $f' = g'' > 0$
$\therefore$ g is decreasing & concave up on $(-4,-3)$.

(c) Where does g have a local maximum value? Justify your answer.

g has a local max at $x = \frac{3}{2}$ b/c $g' = f$ changes signs ⊕ to ⊖.

(d) Where does g have a local minimum value? Justify your answer.

g has a local minimum at $x = -3$ b/c $g' = f$ changes signs ⊖ to ⊕.

(e) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

ABS MAX may be at local max $x = \frac{3}{2}$ or at endpts. $x = -4, 3$

$g(-4) = \int_0^{-4} f(t)\,dt = -\frac{9\pi}{4} + \frac{\pi}{4} = -2\pi$ $g(\frac{3}{2}) = \int_0^{3/2} f(t)\,dt = \frac{9}{4}$ $g(3) = 0$

$\therefore$ ABSOLUTE MAX occurs at $x = 3/2$ when $g(3/2) = 9/4$

(f) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

g has possible inflection pts when
$f' = g'' = 0$ which never happens on $(-4,3)$.
$f' = g''$ is undefined $x = -3$ or $x = 0$.

No Inf Pt at $x = -3$ $f' = g''$ does not change signs
Yes Inf Pt at $x = 0$ b/c $f' = g''$ changes signs.

2013 AP FRQ #2 **Calculator Active**

A particle moves along a straight line. For time, $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{\frac{6}{5}} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

1. Find all the values of t on the interval $2 \le t \le 4$, for which the speed of the particle is 2.

$Y_1 = |v(t)|$
$Y_2 = 2$

$|v(t)| = 2$
$t = 3.127$ or 3.128
$t = 3.473$

2. Write an expression involving an integral that gives the position, $s(t)$. Use this expression to find the position of the particle at time, $t = 5$.

$s(t) = s(0) + \int_0^t v(t)\,dt$
$s(5) = 10 + \int_0^5 v(t)\,dt$
$s(5) = -9.207$

3. Find all times, t, on the interval $0 \le t \le 5$, at which the particle changes direction. Justify your answer.

$v(t) = 0$ @ $t = 0.536$ & 3.317 or 3.318

$v(t)$ changes signs from ⊖ to ⊕ at $t = 0.536$ ▶ $\boxed{A}$
$v(t)$ changes signs from ⊕ to ⊖ at $t = 3.317$. ▶ $\boxed{B}$
Therefore the particle changes direction at $t = 0.536$ & 3.317.

4. Is the speed of the particle increasing or decreasing at time, $t = 4$? Justify your answer.

$v(4) = -11.475$ or $-11.476 < 0$
$a(4) = v'(4) = -22.295$ or $-22.296 < 0$
Speed is increasing at $t = 4$ b/c $v(4) < 0$ & $a(4) < 0$.

5. When is the particle furthest from the origin on the time interval $0 \le t \le 5$? Show the work that leads to your answer.

$s(0) = 10$
$s(0.536) = s(0) + \int_0^{0.536} v(t)\,dt = 9.4025$ ↙A
$s(3.317) = s(0) + \int_0^{3.317} v(t)\,dt = 20.0381$ ↙B
$s(5) = s(0) + \int_0^5 v(t)\,dt = -9.2073$

The particle is furthest from the origin at time $t = 3.317$.

Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) Solve $\lvert v(t) \rvert = 2$ on $2 \le t \le 4$. $t = 3.128$ (or 3.127) and $t = 3.473$	$2 : \begin{cases} 1 : \text{considers } \lvert v(t) \rvert = 2 \\ 1 : \text{answer} \end{cases}$
(b) $s(t) = 10 + \int_0^t v(x)\,dx$ $s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$	$2 : \begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$
(c) $v(t) = 0$ when $t = 0.536033,\ 3.317756$ $v(t)$ changes sign from negative to positive at time $t = 0.536033$. $v(t)$ changes sign from positive to negative at time $t = 3.317756$. Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$
(d) $v(4) = -11.475758 < 0,\ a(4) = v'(4) = -22.295714 < 0$ The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.	2 : conclusion with reason

Motion Key p.11

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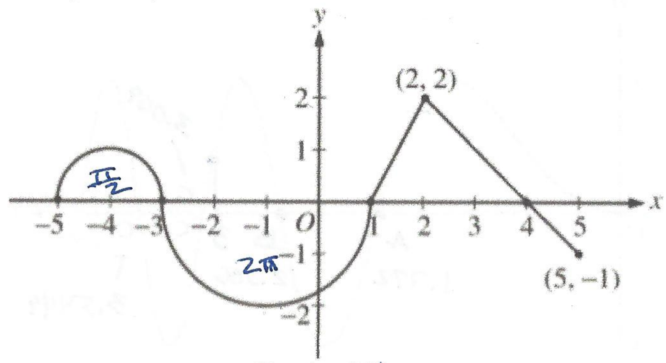
2007B AP FRQ #4 **Non-Calculator**



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

$f'(x) = 0$ at $x = -3, 1, 4$
f' changes signs ⊕ to ⊖ at $x = -3$ & 4 ∴ f has relative maximum at $x = -3$ & 4.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

f will have inflection points when f'' changes signs which occurs when f' changes from increasing to decreasing or decreasing to increasing & this occurs at $x = -4, -1, 2$.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

f is concave up and has positive slope when f' is increasing and f' is positive which occurs on $(-5, -4)$, $(1, 2)$.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

ABS min occur at rel min ∴ $x = 1$ when f' changes sign ⊖ to ⊕ or at endpoints $x = -5, 5$

$f(-5) = f(1) + \int_1^{-5} f'(x)\,dx = 3 + 2\pi - \frac{\pi}{2} = 3 + \frac{3\pi}{2} = \frac{6+3\pi}{2} > 3$

$f(5) = f(1) + \int_1^5 f'(x)\,dx = 3 + 3 - \frac{1}{2} = 5.5 > 3$

$f(1) = f(1) + \int_1^1 f'(x)\,dx = 3 + 0 = 3$

y-value

∴ ABS MIN is $f(1) = 3$ on $[-5, 5]$.

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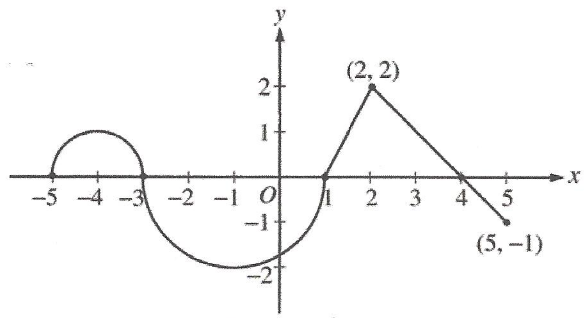
Question 4

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$
f' changes from positive to negative at -3 and 4.
Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$2: \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1$, and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1$, and 2.

$2: \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$2: \begin{cases} 1 : \text{intervals} \\ 1 : \text{explanation} \end{cases}$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$3: \begin{cases} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{cases}$

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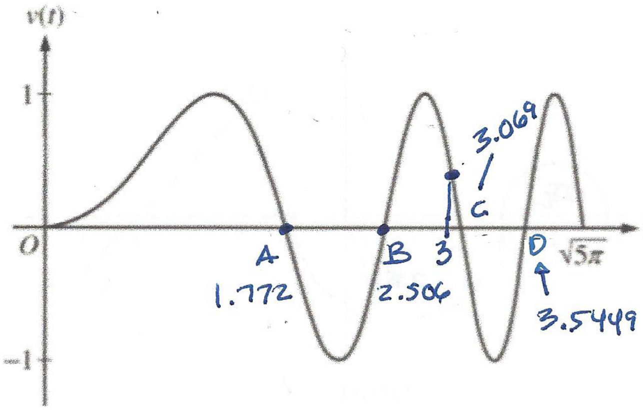
2007B AP FRQ #2

Calculator Active



$v(t) = \sin(t^2)$

2. A particle moves along the x-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \le t \le \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.

$a(3) = v'(t) = 2t\cos(t^2)\Big|_{t=3} = 6\cos 9 = \underline{-5.466}$ or $\underline{-5.467}$

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

OPTION 1
$v(t) = 0 \rightarrow A = 1.772 \quad B = 2.506$
$\int_0^A v(t)\,dt + \int_A^B |v(t)|\,dt + \int_B^3 v(t)\,dt = \underline{1.702}$ = Total distance

* OPTION 2
Total Distance $= \int_0^3 |v(t)|\,dt = \underline{1.702}$

(c) Find the position of the particle at time $t = 3$.

$x(3) = x(0) + \int_0^3 v(t)\,dt$

$x(3) = 5 + .77356\ldots$

$x(3) = \underline{5.773 \text{ or } 5.774}$

(d) For $0 \le t \le \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

The particle moves right on $(0, A) = (0, 1.772)$
on $(B, C) = (2.506, 3.069)$
on $(D, \sqrt{5\pi}) = (3.549, \sqrt{5\pi})$
b/c $v(t) > 0$.

So the particle will be furthest right at $t = 1.772$ or 3.069 or $\sqrt{5\pi}$. (3.963)

$x(A) = 5 + \int_0^A v(t)\,dt = \underline{5.8948}$ ← ∴ Particle is furthest right at $t = A = 1.772$.

$x(C) = 5 + \int_0^C v(t)\,dt = 5.7882$

$x(\sqrt{5\pi}) = 5 + \int_0^{\sqrt{5\pi}} v(t)\,dt$ 5.7524

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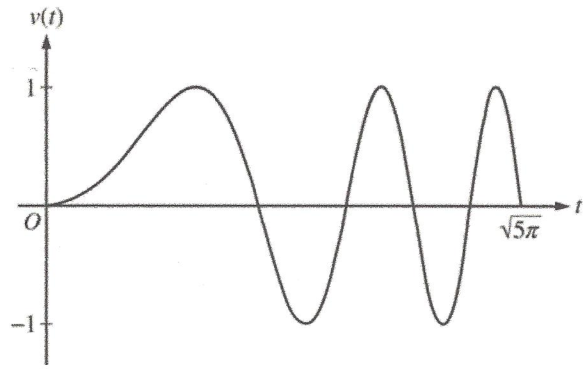
Question 2

A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.



(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

(a) $a(3) = v'(3) = 6\cos 9 = -5.466$ or -5.467

$1 : a(3)$

(b) Distance $= \int_0^3 |v(t)|\,dt = 1.702$

OR

For $0 < t < 3$, $v(t) = 0$ when $t = \sqrt{\pi} = 1.77245$ and $t = \sqrt{2\pi} = 2.50663$

$x(0) = 5$

$$x(\sqrt{\pi}) = 5 + \int_0^{\sqrt{\pi}} v(t)\,dt = 5.89483$$

$$x(\sqrt{2\pi}) = 5 + \int_0^{\sqrt{2\pi}} v(t)\,dt = 5.43041$$

$$x(3) = 5 + \int_0^3 v(t)\,dt = 5.77356$$

$$|x(\sqrt{\pi}) - x(0)| + |x(\sqrt{2\pi}) - x(\sqrt{\pi})| + |x(3) - x(\sqrt{2\pi})| = 1.702$$

$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(c) $x(3) = 5 + \int_0^3 v(t)\,dt = 5.773$ or 5.774

$3 : \begin{cases} 2 \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = 5 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The particle's rightmost position occurs at time $t = \sqrt{\pi} = 1.772$.

The particle changes from moving right to moving left at those times t for which $v(t) = 0$ with $v(t)$ changing from positive to negative, namely at $t = \sqrt{\pi}, \sqrt{3\pi}, \sqrt{5\pi}$ ($t = 1.772, 3.070, 3.963$).

Using $x(T) = 5 + \int_0^T v(t)\,dt$, the particle's positions at the times it changes from rightward to leftward movement are:

T:	0	$\sqrt{\pi}$	$\sqrt{3\pi}$	$\sqrt{5\pi}$
$x(T)$:	5	5.895	5.788	5.752

The particle is farthest to the right when $T = \sqrt{\pi}$.

$3 : \begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$