AREAS & VOLUMES **AP REVIEW**

Multiple Choice

Identify the choice that best completes the statement or answers the question.

NON-CALCULATOR: # 1, 2, 6, 8 & CALCULATOR ACTIVE: # 3, 4, 5, 7, 9, 10, 11

1. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is:

NC

a. $\frac{14}{3}$ b. $\frac{16}{3}$ c. $\frac{28}{3}$ d. $\frac{32}{3}$ e. 8π

2. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

NC

a. $\frac{2}{3}$ b. $\frac{8}{3}$ c. 4 d. $\frac{14}{3}$ e. $\frac{16}{3}$

3. Let R be the region enclosed by the graph of $y = 1 + \ln(\cos^4 x)$ the x-axis, the lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is:

C

a. 0 b. 1 c. 2 d. 3 e. 4

4. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$ and the y-axis?

C

a. 0.127 b. 0.385 c. 0.400 d. 0.600 e. 0.947

5. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$ ______.

C

a. 1.471 b. 1.414 c. 1.277 d. 1.120 e. 0.436

6. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is:

NC

a. $\frac{32\pi}{5}$ b. $\frac{16\pi}{3}$ c. $\frac{16\pi}{5}$ d. $\frac{8\pi}{3}$ e. π

7. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is:

C

a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. 1 d. 2 e. $\frac{1}{3}\left(e^3 - 1\right)$

8. The base of a solid is the region in the first quadrant enclosed by the graph of y = 2 – x2 and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by:

NC

a. $\pi\int_0^2 (2-y)^2\,dy$ b. $\int_0^2 (2-y)\,dy$ c. $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$ d. $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$ e. $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$

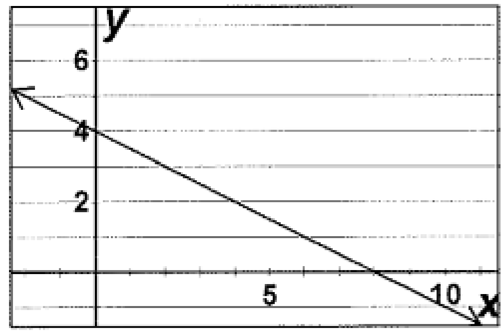
9. The region bounded by the graph of $y = 2x - x^2$ and the x-axis is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. What is the volume of the solid?

C

a. 1.333 b. 1.067 c. 0.577 d. 0.462 e. 0.267

10. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure at the right. If cross sections of the solid perpendicular to the x-axis are semi-circles, what is the volume of the solid?

C



a. 12.566 b. 14.661 c. 16.755

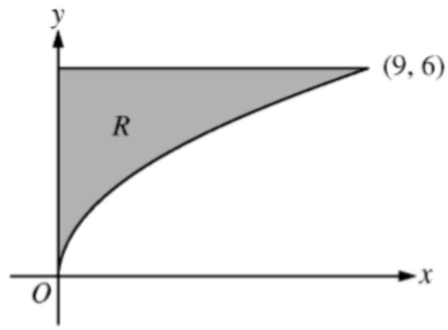
d. 67.021 e. 134.041

11. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

C

a. 2.561 b. 6.612 c. 8.046 d. 8.755 e. 20.773

FRQ – NON-CALCULATOR



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

AVERAGE & INSTANTANEOUS RATES OF CHANGE AP REVIEW

Multiple Choice

Identify the choice that best completes the statement or answers the question.

NON-CALCULATOR

1. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2-2}{x-1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

CALCULATOR

2. Let f be a function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701 (B) -0.567 (C) -0.391 (D) -0.302 (E) -0.258

3. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[1, 4]$?

(A) 0.456 (B) 1.244 (C) 2.164 (D) 2.342 (E) 2.452

4. Let f be a function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

5. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

FRQ – CALCULATOR ACTIVE

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

FRQ – NON-CALCULATOR

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\,dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\,dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

BIG THEOREMS **AP REVIEW**

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. The function f is continuous on the closed interval [0, 2] and has values that are given in the table ~~above~~. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k=$

x	0	1	2
$f(x)$	1	k	2

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. 3

____ 2. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c)=3$.

a. None
b. I only
c. I and II only
d. I and III only
e. I, II and III

____ 3. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

a. There exists c, where $-2 < c < 1$, such that $f(c) = 0$.
b. There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.
c. There exists c, where $-2 < c < 1$, such that $f(c) = 3$.
d. There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.
e. There exists c, where $-2 \le c \le 1$ such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

____ 4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

a. $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.
b. $f'(c) = 0$ for come c such that $a < c < b$.
c. f has a minimum value on $a \le x \le b$.
d. f has a maximum value on $a \le x \le b$.
e. $\int_a^b f(x)dx$ exists.

____ 5. The function f is continuous and differentiable on the closed interval [0,4]. The table ~~above~~ gives selected values of f on this interval. Which of the following statements must be true?

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

a. The minimum value of f on [0, 4] is 2.
b. The maximum value of f on [0, 4] is 4.
c. $f(x) > 0$ for $0 < x < 4$
d. $f'(x) < 0$ for $2 < x < 4$
e. There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

____ 6. Let $f(x) = \int_0^{x^2} \sin t\, dt$. At how many points in the closed interval $[0, \sqrt{\pi}]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

a. Zero b. One c. Two d. Three e. Four

____ 7. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1,4]?

a. 0.456 b. 1.244 c. 2.164 d. 2.342 e. 2.452

____ 8. If f is the antiderivative of $\dfrac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

a. -0.012 b. 0 c. 0.016 d. 0.376 e. 0.629

____ 9. If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

a. -3 b. -2 c. 2 d. 3 e. 18

____ 10. $\dfrac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)dt\right) =$

a. $-\cos\left(x^6\right)$ b. $\sin\left(x^3\right)$ c. $\sin\left(x^6\right)$ d. $2x\sin\left(x^3\right)$ e. $2x\sin\left(x^6\right)$

_____ 11. $\lim_{x \to 1} \dfrac{\int_1^x e^{t^2}\,dt}{x^2 - 1}$ is

a. 0   b. 1   c. $\frac{e}{2}$ d. e e. nonexistent

FRQ – NON-CALCULATOR

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

6. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\,dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\,dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_0^{30} a(t)\,dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t)\,dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

DIFFERENTIAL EQUATIONS **AP REVIEW**

Multiple Choice

Identify the choice that best completes the statement or answers the question.

_____ 1. Population y grows according to the equation $\frac{dy}{dt} = ky$ where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is:

a. 0.069   b. 0.200   c. 0.301   d. 3.322   e. 5.000

_____ 2. If $\frac{dy}{dx} = ky$ and k is a nonzero constant, then y could be:

a. $2e^{kty}$ b. $2e^{kt}$ c. $e^{kt} + 3$ d. $kty + 5$ e. $\frac{1}{2}ky^2 + \frac{1}{2}$

_____ 3. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

a. −1   b. $-\frac{1}{3}$ c. 0   d. $\frac{1}{3}$ e. 1

_____ 4. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

a. $V(t) = k\sqrt{t}$ b. $V(t) = k\sqrt{V}$ c. $\dfrac{dV}{dt} = k\sqrt{t}$ d. $\dfrac{dV}{dt} = \dfrac{k}{\sqrt{V}}$ e. $\dfrac{dV}{dt} = k\sqrt{V}$

_____ 5. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

a. $y = 5x - 3$ b. $y = x^2 + 1$ c. $y = x^2 + 3x$ d. $y = x^2 + 3x - 2$ e. $y = x^2 + 3x - 3$

_____ 6. The slope field shown at right is for which of the following differential equations?

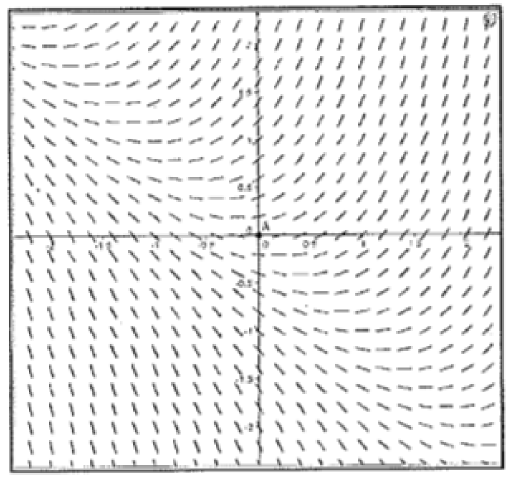
a. $\dfrac{dy}{dx} = 1 + x$

b. $\dfrac{dy}{dx} = x^2$

c. $\dfrac{dy}{dx} = x + y$

d. $\dfrac{dy}{dx} = \dfrac{x}{y}$

e. $\dfrac{dy}{dx} = \ln y$



_____ 7. The slope field shown at right is for which of the following differential equations?

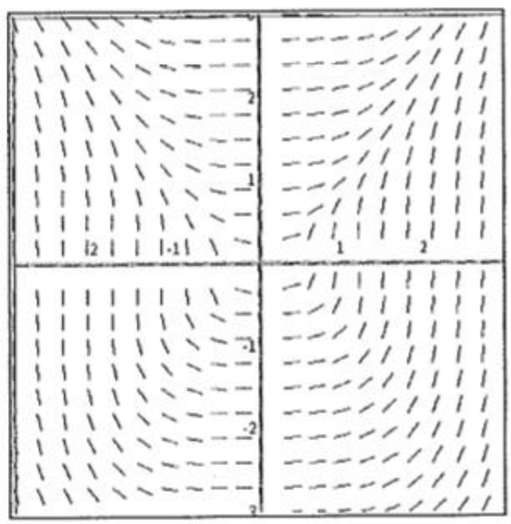
a. $\dfrac{dy}{dx} = \dfrac{x}{y}$

b. $\dfrac{dy}{dx} = \dfrac{x^2}{y^2}$

c. $\dfrac{dy}{dx} = \dfrac{x^3}{y}$

d. $\dfrac{dy}{dx} = \dfrac{x^2}{y}$

e. $\dfrac{dy}{dx} = \dfrac{x^3}{y^2}$



FRQ – NON-CALCULATOR

6. Solutions to the differential equation $\dfrac{dy}{dx} = xy^3$ also satisfy $\dfrac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\dfrac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. $\int_0^{\frac{\pi}{4}} \sin x\,dx =$

a. $-\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{2}}{2}$ c. $-\frac{\sqrt{2}}{2}-1$ d. $-\frac{\sqrt{2}}{2}+1$ e. $\frac{\sqrt{2}}{2}-1$

____ 2. $\int_0^1 e^{-4x}\,dx =$

a. $\frac{-e^{-4}}{4}$
b. $-4e^{-4}$
c. $e^{-4}-1$
d. $\frac{1}{4}-\frac{e^{-4}}{4}$
e. $4-4e^{-4}$

____ 3. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

a. $e^{-t}+C$
b. $e^{-\frac{t}{2}}+C$
c. $e^{\frac{t}{2}}+C$
d. $2e^{\frac{t}{2}}+C$
e. $e^{t}+C$

____ 4. $\int x^2\cos\left(x^3\right)dx =$

a. $-\frac{1}{3}\sin\left(x^3\right)+C$
b. $\frac{1}{3}\sin\left(x^3\right)+C$
c. $-\frac{x^3}{3}\sin\left(x^3\right)+C$
d. $\frac{x^3}{3}\sin\left(x^3\right)+C$
e. $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

____ 5. Using the substitution $u=2x+1$, $\int_0^2\sqrt{2x+1}\,dx$ is equivalent to

a. $\frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\sqrt{u}\,du$
b. $\frac{1}{2}\int_0^2\sqrt{u}\,du$
c. $\frac{1}{2}\int_1^5\sqrt{u}\,du$
d. $\int_0^2\sqrt{u}\,du$
e. $\int_1^5\sqrt{u}\,du$

____ 6. $\int_1^e \left(\frac{x^2-1}{x}\right)dx =$

a. $e-\frac{1}{e}$
b. e^2-e
c. $\frac{e^2}{2}-e+\frac{1}{2}$
d. e^2-2
e. $\frac{e^2}{2}-\frac{3}{2}$

____ 7. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

a. -3
b. 0
c. 3
d. -3 and
e. 3-3, 0, and 3

____ 8. $\int_1^2 \left(4x^3-6x\right)dx =$

a. 2
b. 4
c. 6
d. 36
e. 42

____ 9. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval [0, 2] ?

a. $\frac{26}{9}$ b. $\frac{52}{9}$ c. $\frac{26}{3}$ d. $\frac{52}{3}$ e. 24

____ 10. If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1)=0$, then $f(4) =$

a. -0.012
b. 0
c. 0.016
d. 0.376
e. 0.629

____ 11. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9) =$

a. 0.048
b. 0.144
c. 5.827
d. 23.308
e. 1,640.250

FRQ – NON-CALCULATOR

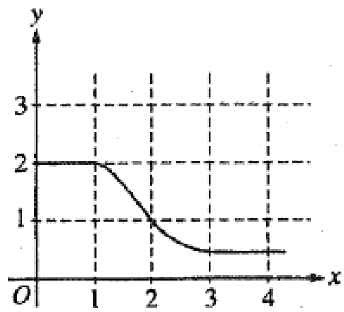
6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

Multiple Choice

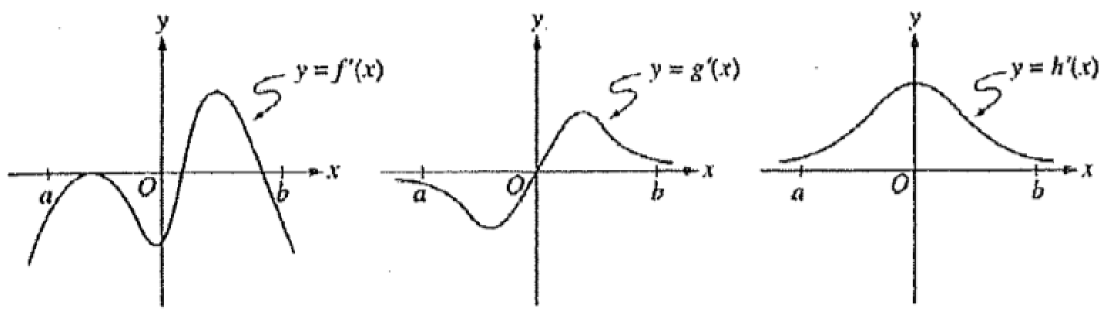
Identify the choice that best completes the statement or answers the question.

1. 

The graph of f is shown in the figure.

If $\int_1^3 f(x)dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

a. 0.3 b. 1.3 c. 3.3 d. 4.3 e. 5.3

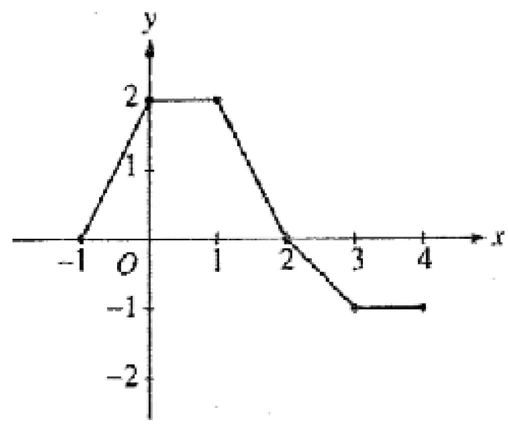


2. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

a. f only b. g only c. h only d. f and g only e. f, g, and h

3. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown. What is the value of $\int_{-1}^{4} f(x)dx$?

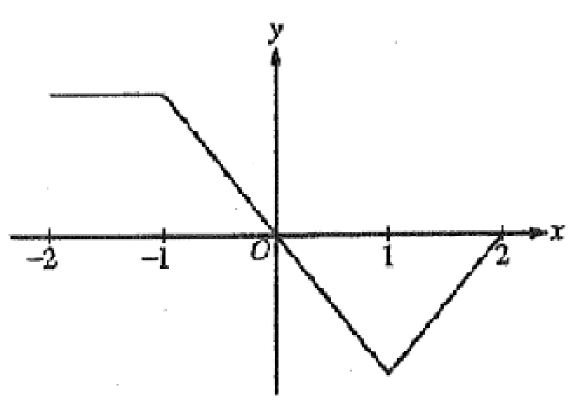
a. 1
b. 2.5
c. 4
d. 5.5
e. 8



4. The graph of f', the derivative of the function f, is shown

Which of the following statements is true about f?

a. f is decreasing for $-1 \le x \le 1$.
b. f is increasing for $-2 \le x \le 0$.
c. f is increasing for $1 \le x \le 2$.
d. f has a local minimum at $x = 0$.
e. f is not differentiable at $x = -1$ and $x = 1$.



Graph of f'

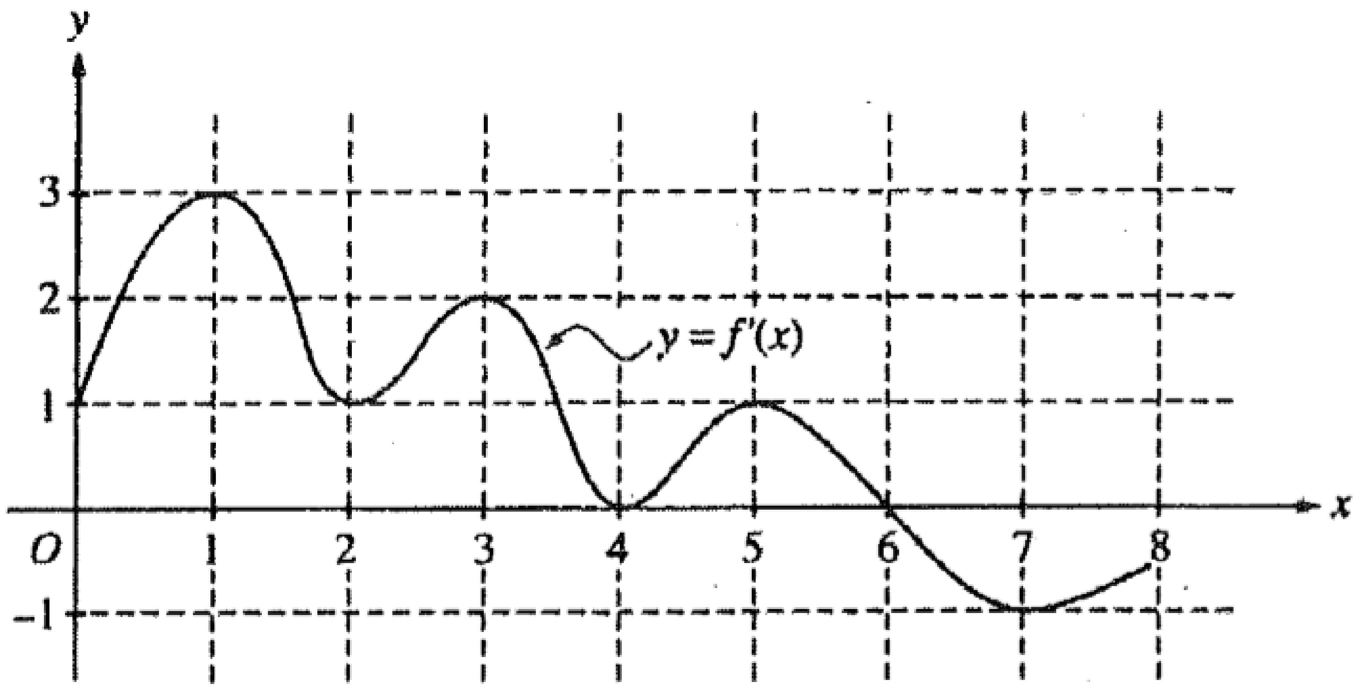
5. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval (0, 10)?

a. One b. Three c. Four d. Five e. Seven

6. The function f has first derivative given by $f'(x) = \frac{\sqrt{x}}{1+x+x^3}$ What is the x-coordinate ofthe inflection point of the graph of f?

a. 1.008 b. 0.473 c. 0 d. -0.278 e. The graph of f has no inflection point.

Questions 7-9 refer to the graph and the information given below.



The function f is defined on the closed interval [0, 8]. The graph of its derivative f' is shown above.

7. The point (3, 5) is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at (3, 5) is

a. $y = 2$ b. $y = 5$ c. $y - 5 = 2(x - 3)$ d. $y + 5 = 2(x - 3)$ e. $y + 5 = 2(x + 3)$

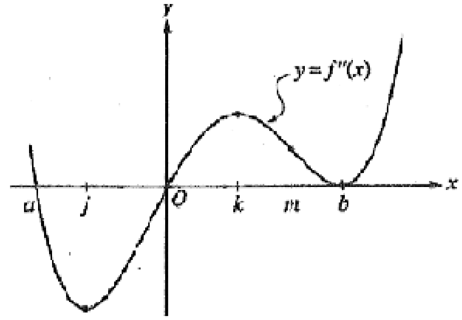
8. How many points of inflection does the graph of f have?

a. Two b. Three c. Four d. Five e. Six

9. At what value of x does the absolute minimum of f occur?

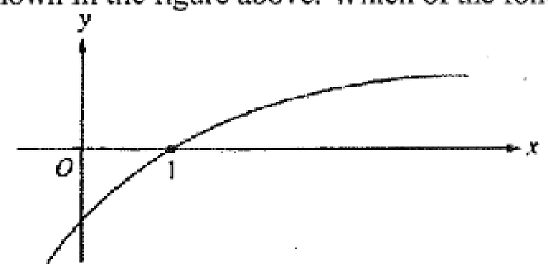
a. 0 b. 2 c. 4 d. 6 e. 8

10. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$ The graph of f'' is shown ~~above~~. For what values of x does the graph of f have a point of inflection?



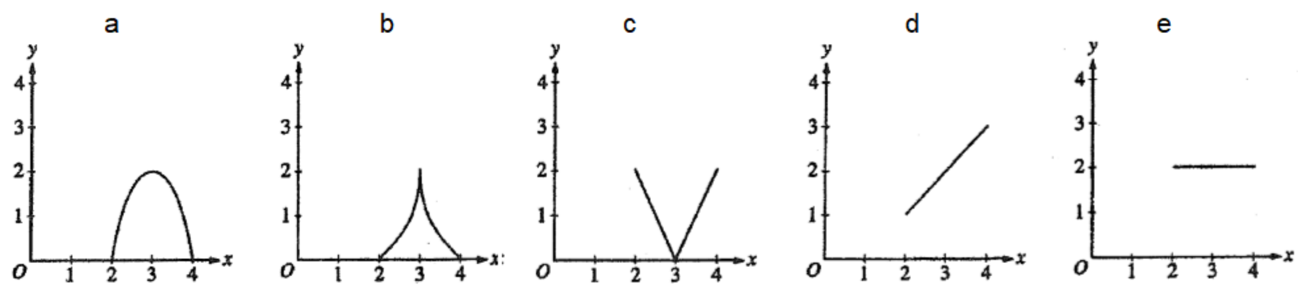
a. 0 and a only
b. 0 and m only
c. b and j only
d. 0, a, and b
e. b, j, and k

11. The graph of a twice-differentiable function f is shown In the figure ~~above~~. Which of the following is true?



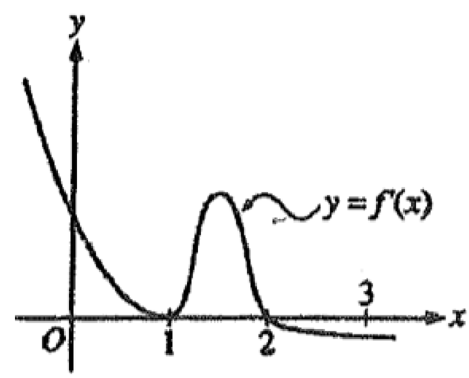
a. $f(1) < f'(1) < f''(1)$
b. $f(1) < f''(1) < f'(1)$
c. $f'(1) < f(1) < f''(1)$
d. $f''(1) < f(1) < f'(1)$
e. $f''(1) < f'(1) < f(1)$

12. On the closed interval [2, 4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$?



13. The graph of f', the derivative of the function f, is shown ~~above.~~ If $f(0) = 0$, which of the following must be true?

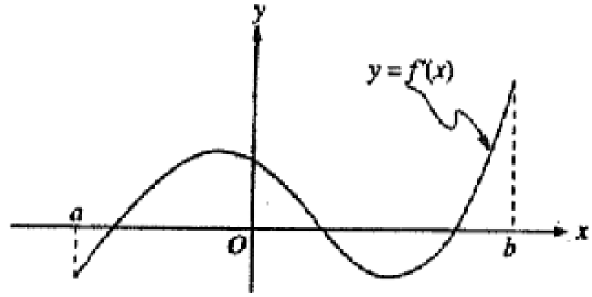
I. $f(0) > f(1)$
II. $f(2) > f(1)$
III. $f(1) > f(3)$

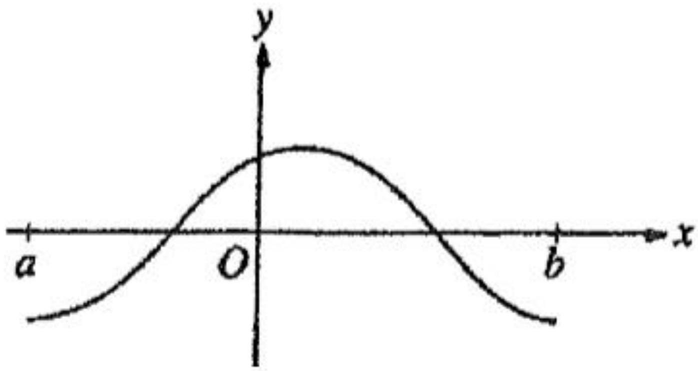


a. I only
b. II only
c. III only
d. I and II only
e. II and III only

14. The graph of f', the derivative of f, is shown in the figure ~~above~~. Which of the following describes all relative extrema of f on the open interval (a, b) ?

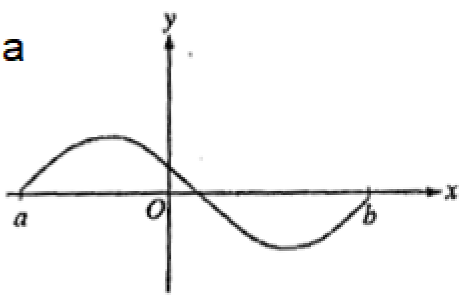
a. One relative maximum and two relative minima
b. Two relative maxima and one relative minimum
c. Three relative maxima and one relative minimum
d. One relative maximum and three relative minima
e. Three relative maxima and two relative minima



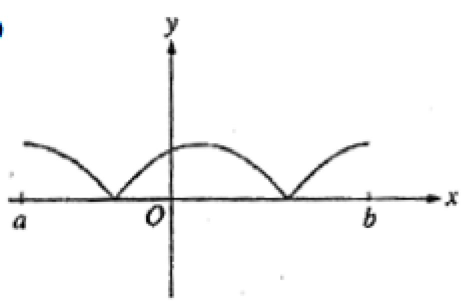


15. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

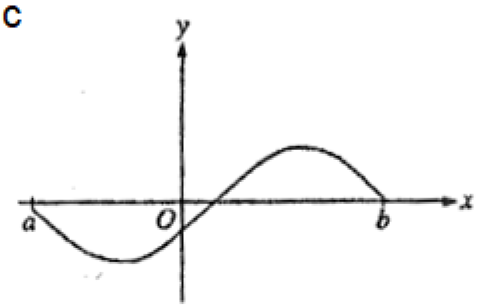
a



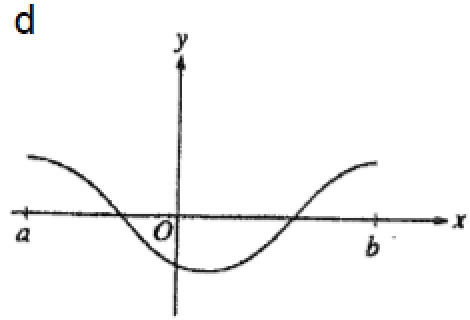
b



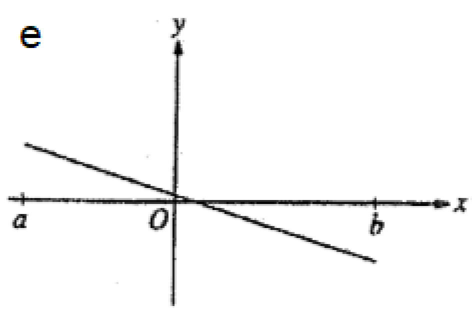
c



d

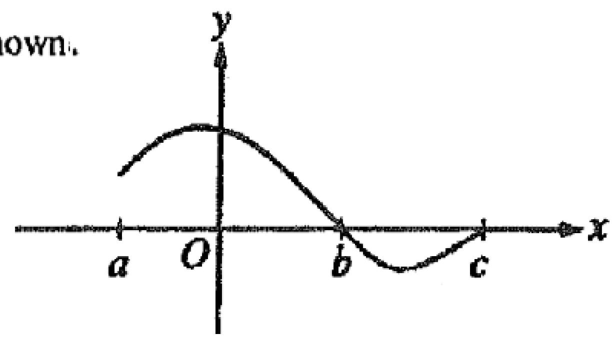


e

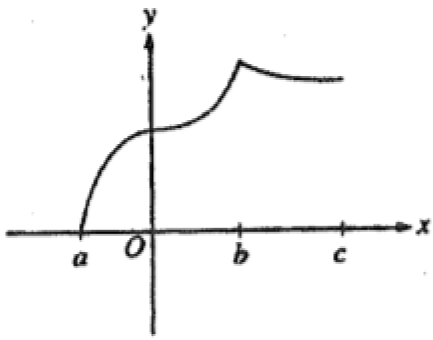


16. Let $f(x) = \int_a^x h(t)dt$, where h has the graph shown.

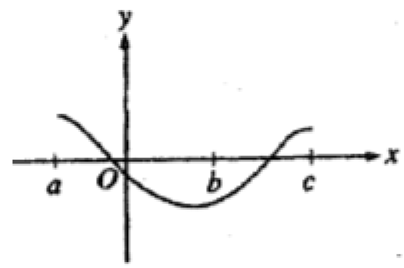
Which of the following could be the graph of f?



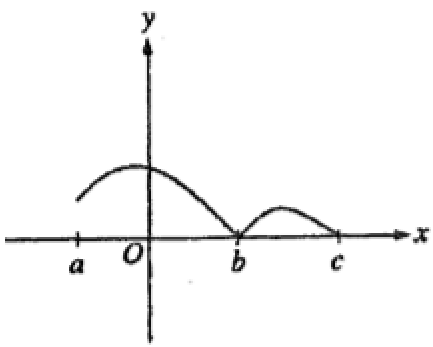
a



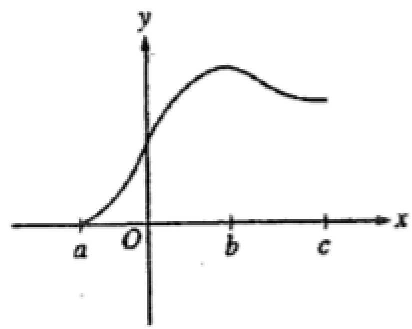
d



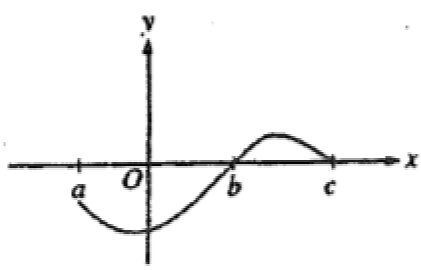
b



e



c



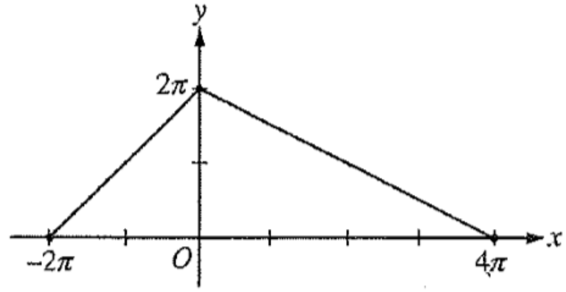
FRQ – NON-CALCULATOR

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

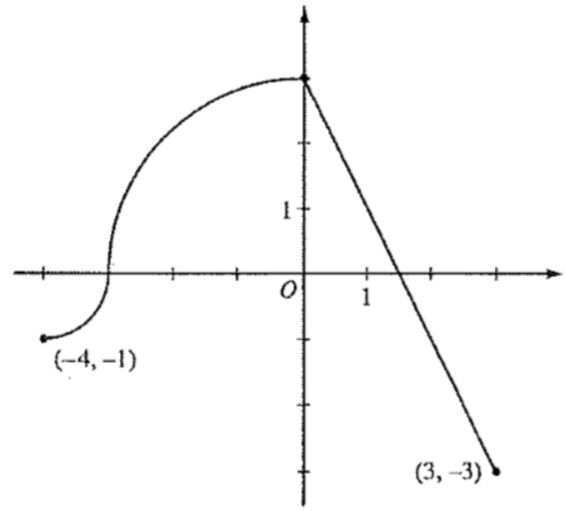
(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.



Graph of g

FRQ – NON-CALCULATOR

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.



Graph of f

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

PARTICLE MOTION (12 MC & 1 FRQ) AP REVIEW

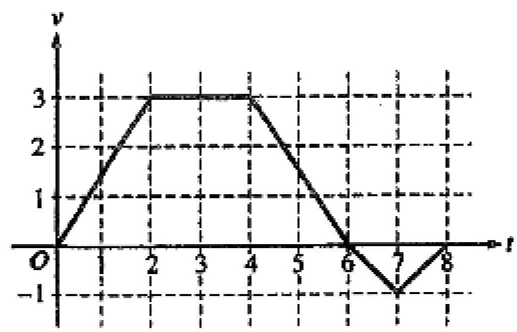
Multiple Choice

Identify the choice that best completes the statement or answers the question.

NON-CALCULATOR #1-6 & Calculator Active #7-12

Questions 1 - 2 refer to the following situation.

A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown.



____ 1. At what value of t does the bug change direction?

a. 2
b. 4
c. 6
d. 7
e. 8

____ 2. What is the total distance the bug traveled from $t=0$ to $t=8$?

a. 14
b. 13
c. 11
d. 8
e. 6

____ 3. A particle moves along the x-axis so that its position at time t is given by $x(t)=t^2-6t+5$. For what value of t is the velocity of the particle zero?

a. 1
b. 2
c. 3
d. 4
e. 5

____ 4. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t)=t^3-3t^2+12t+4$ is

a. 9
b. 12
c. 14
d. 21
e. 40

____ 5. A particle moves along the x-axis so that its acceleration at any time t is $a(t)=2t-7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

a. 0
b. 1
c. 2
d. 3
e. 4

____ 6. A particle moves along the x-axis so that at time $t \ge 0$ its position is given by $x(t)=2t^3-21t^2+72t-53$. At what time t is the particle at rest?

a. $t=1$ only
b. $t=3$ only
c. $t=\frac{7}{2}$ only
d. $t=3$ and $t=\frac{7}{2}$
e. $t=3$ and $t=4$

Calculator Active #7-12

____ 7. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table ~~above~~. If the velocity at $t=0$ is 11 feet per second, the approximate value of the velocity at $t=6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

a. 26 ft/sec
b. 30 ft/sec
c. 37 ft/sec
d. 39 ft/sec
e. 41 ft/sec

_____ 8. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

a. 1.02
b. 1.48
c. 1.85
d. 2.81
e. 3.14

_____ 9. Aparticle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time t =4?

a. -2.016
b. -0.677
c. 1.633
d. 1.814
e. 2.978

_____ 10. The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

a. Zero
b. Three
c. Five
d. Six
e. Seven

_____ 11. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos\left(2 - t^2\right)$. The position of the particle is 3 at time t =0. What is the position of the particle when its velocity is first equal to 0?

a. 0.411
b. 1.310
c. 2.816
d. 3,091
e. 3.411

_____ 12. The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{\frac{3}{2}} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

a. 2.545 meters
b. 10.263 meters
c. 34.125 meters
d. 54.889 meters
e. 89.005 meters

Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

RATES & ACCUMULATIONS **AP REVIEW**

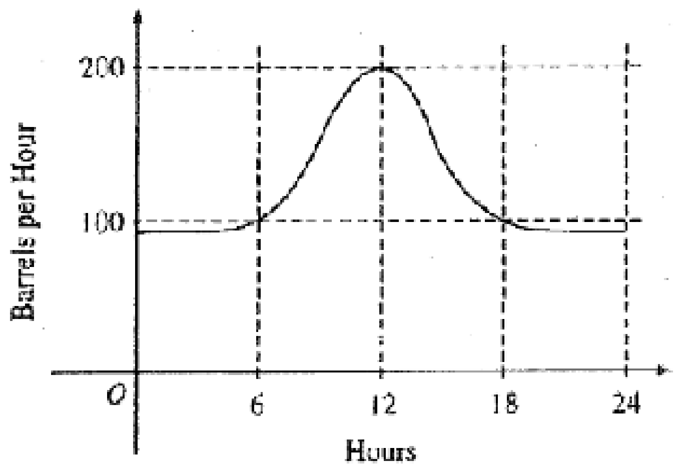
Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 2. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown ~~above~~. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

a. 500 b. 600 c. 2,400

d. 3,000 e. 4,800



____ 4. Insects destroyed a crop at the rate of $\frac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

a. 125 b. 100 c. 88 d. 50 e. 12

____ 5. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

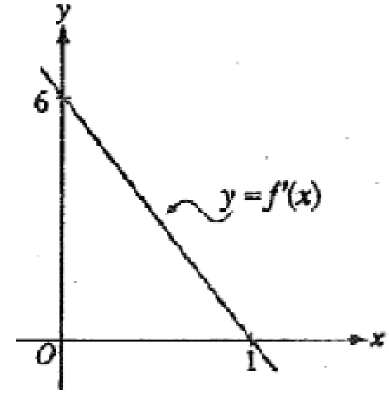
a. $\int_{1.572}^{3.514} r(t)\,dt$ b. $\int_{0}^{8} r(t)\,dt$ c. $\int_{0}^{2.667} r(t)\,dt$ d. $\int_{1.572}^{3.514} r'(t)\,dt$ e. $\int_{0}^{2.667} r'(t)\,dt$

____ 6. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

a. 112°F b. 119°F c. 147°F d. 238°F e. 335°F

_____ 7. The graph of f', the derivative of f, is the line shown in the figure If $f(0) = 5$. then $f(1) =$

a. 0
b. 3
c. 6
d. 8
e. 11



_____ 8. A particle moves along the x-axis so that at any time $t > 0$. its acceleration is given by $a(t) = \ln\left(1 + 2^t\right)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

a. 0.462 b. 1.690 c. 2.555 d. 2.886 e. 3.346

FRQ – CALCULATOR ACTIVE

2 A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

RELATED RATES (5 MC & 1 FRQ) AP REVIEW

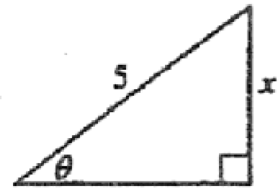
Multiple Choice

Identify the choice that best completes the statement or answers the question.

_____ 1. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

a. 57.60
b. 57.88
c. 59.20
d. 60.00
e. 67.40

_____ 2. In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?



a. 3
b. $\frac{15}{4}$
c. 4
d. 9
e. 1

_____ 3. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

a. $-(0.2)\pi C$
b. $-(0.1)C$
c. $-\frac{(0.1)C}{2\pi}$
d. $(0.1)^2 C$
e. $(0.1)^2 \pi C$

_____ 4. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

a. A is always increasing.
b. A is always decreasing.
c. A is decreasing only when $b < h$.
d. A is decreasing only when $b > h$.
e. A remains constant.

_____ 5. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

a. 0.04π m²/sec

b. 0.4π m²/sec

c. 4π m²/sec

d. 20π m²/sec

e. 100π m²/sec

Related Rates FRQ

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

   (a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

   (b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

   (c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

TABLE QUESTIONS **AP REVIEW**

Multiple Choice

Identify the choice that best completes the statement or answers the question.

NON-CALCULATOR #1-3 & Calculator Active #4-9

1.

x	2	5	7	8
$f(x)$	10	30	40	20

The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

a. 110
b. 130
c. 160
d. 190
e. 210

2.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
−1	6	5	3	−2
1	3	−3	−1	2
3	1	−2	2	3

The table above gives values of f, f', g, and g' at selected values of x. If $h(x) = f(g(x))$, then $h'(1) =$

a. 5
b. 6
c. 9
d. 10
e. 12

3. For all x in the closed interval [2, 5], the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

a.

x	$f(x)$
2	7
3	9
4	12
5	16

b.

x	$f(x)$
2	7
3	11
4	14
5	16

c.

x	$f(x)$
2	16
3	12
4	9
5	7

d.

x	$f(x)$
2	16
3	14
4	11
5	7

e.

x	$f(x)$
2	16
3	13
4	10
5	7

Calculator Active #4-9

4.

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)dx$?

a. 8
b. 12
c. 16
d. 24
e. 32

5.

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at t =0 is 11 feet per second, the approximate value of the velocity at t = 6, computed using a left-hand Riemann sum with three subintervals of equal length, is

a. 26 ft/sec
b. 30 ft/sec
c. 37 ft/sec
d. 39 ft/sec
e. 41 ft/sec

6.

x	2	5	10	14
$f(x)$	12	28	34	30

The function f is continuous on the closed interval [2, 14] and has values as shown in the table above. Using the subintervals [2, 5], [5,10], and [10, 14], what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?

a. 296
b. 312
c. 343
d. 374
e. 390

7.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k=$

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. 3

8.

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

a. $-2 \le x \le 2$ only
b. $-1 \le x \le 1$ only
c. $x \ge -2$
d. $x \ge 2$ only
e. $x \le -2$ or $x \ge 2$

9.

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

The function f is continuous and differentiable on the closed interval [0,4]. The table above gives selected values of f on this interval. Which of the following statements must be true?

a. The minimum value of f on [0, 4] is 2.
b. The maximum value of f on [0, 4] is 4.
c. $f(x) > 0$ for $0 < x < 4$
d. $f'(x) < 0$ for $2 < x < 4$
e. There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

FRQ – Calculator Active

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

FRQ – NON-CALCULATOR

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.