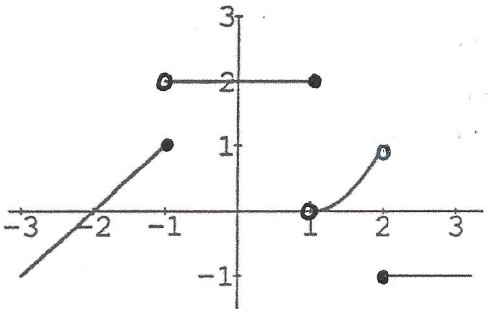
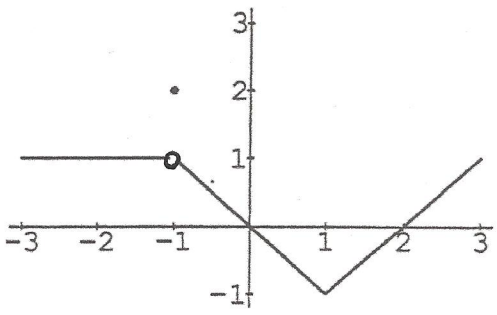
Section 4: Limits

1. The graphs of the functions f and g are given below.



graph of f



graph of g

Determine whether the following limits exist. If they do, then find the limit.

a. $\lim_{x\to -1} f(x)$

b. $\lim_{x\to 1} f(x)$

c. $\lim_{x\to -1} g(x)$

d. $\lim_{x\to 1} g(x)$

e. $\lim_{x\to -1} f(x)+g(x)$

f. $\lim_{x\to 0} 2f(x)+3g(x)$

g. $\lim_{x\to -1} f(x)g(x)$

h. $\lim_{x\to 2} f(x)g(x)$

i. $\lim_{x\to 0} \dfrac{f(x)}{g(x)}$

j. $\lim_{x\to 0} \dfrac{g(x)}{f(x)}$

k. $\lim_{x\to -2} g(f(x))$

l. $\lim_{x\to -1} f(g(x))$

2. The graphs of functions f and g are those given in Problem 1 above. Determine whether the following limits exist and find the limit when it exists.

a. $\lim_{x\to -1^-} f(x)$

b. $\lim_{x\to -1^+} f(x)$

c. $\lim_{x\to -1^-} g(x)$

d. $\lim_{x\to -1^+} g(x)$

e. $\lim_{x\to 0^-} f(x+2)$

f. $\lim_{x\to -1^-} f(x^2)$

Sketching Graphs from LIMITS and Properties of LIMITS Name_____

Sketch the graph of an example of a function f that satisfies all of the given conditions.

1. $f(3)$ exists but $\lim_{x\to 3} f(x)$ does not exist

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2. $f(-2)$ exists, $\lim_{x\to -2^+} f(x) = f(-2)$, but $\lim_{x\to -2} f(x)$ does not exist.

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3. $f(4)$ exists, $\lim_{x\to 4} f(x)$ exists, but f is not continuous at $x = 4$

4.

$\lim_{x\to 0^-} f(x) = 1, \quad \lim_{x\to 0^+} f(x) = -1, \quad \lim_{x\to 2^-} f(x) = 0, \quad \lim_{x\to 2^+} f(x) = 1$

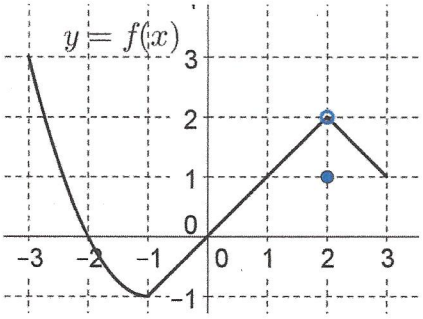
$f(2) = 1, \qquad f(0)$ is undefined

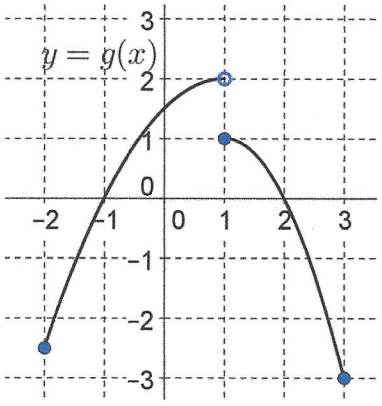
5.

$\lim_{x\to 1} f(x) = 2, \quad \lim_{x\to 5^-} f(x) = \infty, \quad \lim_{x\to 5^+} f(x) = \infty$

$\lim_{x\to \infty} f(x) = -1, \quad \lim_{x\to -2^+} f(x) = -\infty, \quad \lim_{x\to -2^-} f(x) = \infty, \quad \lim_{x\to -\infty} f(x) = 0$

6. The graphs of f and g are given. Use them to evaluate each limit, if it exists. **If the limit does not exist, explain why.**





a. $\lim_{x\to 2} f(x)$

b. $\lim_{x\to 1^+} g(x)$

c. $\lim_{x\to 1} f(x) + g(x)$

d. $\lim_{x\to 2} \dfrac{f(x)}{g(x)}$

e. $\lim_{x\to 0} \dfrac{f(x)}{g(x)}$

f. $\lim_{x\to 1} \sqrt{3 + f(x)}$

PRACTICE ON LIMITS

Continuous or Discontinuous at $x = a$?

Evaluate each limit.

1. $\lim_{x\to 2}\left(4x^2-5x+3\right)=$

2. $\lim_{x\to 0}\sqrt{x^2+4}=$

3. $\lim_{x\to -3}\dfrac{x^2+x-6}{x+3}=$

4. $\lim_{x\to -6}\dfrac{x+6}{x^2+3x-18}=$

5. $\lim_{x\to -2}\dfrac{x^2-4}{x+2}=$

6. $\lim_{x\to 1^-}\dfrac{1}{x-1}=$

7. $\lim_{x\to 1^+}\dfrac{1}{x-1}=$

8. $\lim_{x\to 1}\dfrac{1}{x-1}=$

9. $\lim_{x\to 4^-}\dfrac{|x-4|}{x-4}=$

10. $\lim_{x\to 4^+}\dfrac{|x-4|}{x-4}=$

11. $\lim_{x\to 4}\dfrac{|x-4|}{x-4}=$

12. $h(x)=\begin{cases} x, & x<0 \\ x^2, & 0<x\le 2 \\ 8-x, & x>2 \end{cases}$

a. $\lim_{x\to 0^+} h(x)$

b. $\lim_{x\to 0} h(x)$

c. $\lim_{x\to 1} h(x)$

d. $\lim_{x\to 2^-} h(x)$

e. $\lim_{x\to 2^+} h(x)$

f. $\lim_{x\to 2} h(x)$

WORKSHEET ON LIMIT EVALUATION and ASYMPTOTES

Find the limit. Do not use your calculator.

1. $\lim_{x\to 1^+} \frac{1}{x-1} =$

2. $\lim_{x\to 1} \frac{1}{x-1} =$

3. $\lim_{x\to -3} \frac{1}{(x+3)^2} =$

4. $\lim_{x\to 5} \frac{25-x^2}{5-x} =$

5. $\lim_{x\to 5^-} \frac{1}{(5-x)^2} =$

6. $\lim_{x\to 2} \frac{-1}{(x-2)^2} =$

7. $\lim_{x\to 3} \frac{|x-3|}{x-3} =$

8. $\lim_{x\to 2} [x+1] =$

9. $\lim_{x\to 2^+} \frac{x^3|x-2|}{x-2} =$

10. $\lim_{x\to 4^-} \frac{x^3(x^2-16)}{x-4} =$

11. $f(x) = \begin{cases} x^2-1 \text{ if } x<2 \\ 3x-2 \text{ if } x>2 \end{cases}$

a) $\lim_{x\to 2^-} f(x) =$

b) $\lim_{x\to 2^+} f(x) =$

c) $\lim_{x\to 2} f(x) =$

12. $\lim_{x\to 3^+} \left(x-3-\frac{1}{x-3}\right) =$

13. $g(x) = \begin{cases} x-3 \text{ if } x\neq 1 \\ 4 \text{ if } x=1 \end{cases}$ $\lim_{x\to 1} g(x) =$

14. $h(x) = \begin{cases} x+3 \text{ if } x<1 \\ 3x^2+1 \text{ if } x>1 \end{cases}$ $\lim_{x\to 1} h(x) =$

15. $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4} =$

16. $\lim_{x\to -3} \frac{x^2-3x-18}{x^2-x-12} =$

17. $\lim_{x\to \frac{\pi}{2}^-} \tan x =$

18. $\lim_{x\to 0^-} \cot x =$

On problems 19 - 24:

(a) find $\lim_{x\to\infty} f(x)$

(b) find $\lim_{x\to-\infty} f(x)$

(c) identify all horizontal asymptotes.

Use your graphing calculator on problems 23 and 24.

19. $f(x)=\dfrac{3x^3-x+1}{x+3}$

20. $f(x)=\dfrac{4x^2-3x+5}{2x^3+x-1}$

21. $f(x)=\dfrac{3x+1}{x-4}$

22. $f(x)=\dfrac{3x+1}{|x|+2}$

23. $f(x)=\dfrac{\sin 3x}{x}$

24. $f(x)=\cos\left(\dfrac{1}{x}\right)$

On problems 25 - 28, if necessary use a graphing calculator to

(a) find the vertical asymptotes of $f(x)$

(b) describe the behavior of $f(x)$ to the left and right of each vertical asymptote.

$$\lim_{x\to c-} f(x) \;\&\; \lim_{x\to c+} f(x)$$

25. $f(x)=\dfrac{1}{x^2-4}$

26. $f(x)=\dfrac{x^2+5x+6}{x^2-4}$

27. $f(x)=\dfrac{x^2-2x}{x+1}$

28. $f(x)=\sec x$

Limit Practice #1

Name _____

NO Calculator

Evaluate each limit. If a limit DOES NOT EXIST, tell why.

1. $\lim_{x\to 2} 3x-1$

2. $\lim_{x\to\infty} \frac{1}{x-2}$

3. $\lim_{x\to 3} \frac{x^2-4x+3}{x^2-9}$

4. $\lim_{x\to 4^+} \frac{|x-4|}{x-4}$

5. $\lim_{x\to 5^-} [x]$

6. $\lim_{x\to 6^+} \frac{3x}{x-6}$

7. $\lim_{x\to\infty} \frac{5x^2-1}{2x^2-3x+2}$

8. $\lim_{x\to 3^-} f(x)$ if $f(x)=\begin{cases} 2x-1, & x<3 \\ 4x-2, & x>3 \end{cases}$

9. What are the three conditions that must be met for a function to be continuous at a point?

i.) _____

ii.) _____

iii.) _____

10. Is the function, $h(x)=\begin{cases} \frac{x^2-3x+2}{x^2-4}, & x<2 \\ \frac{3x-5}{4}, & x\geq 2 \end{cases}$ continuous at x = 2? Explain.

11. Given the function $y=\frac{3x^2-3x-6}{x^2-4}$, tell where the function has:

a. Root(s):_____

b. Hole(s):_____

c. Vertical Asymptote(s): _____

d. Horizontal Asymptote(s):_____

12. If $g(x)=\begin{cases} 2x^2-3, & x<2 \\ kx-1, & x>2 \end{cases}$, what value of k will make $g(x)$ continuous at $x=2$?

13. Graph a function that the following list of limits describes.

$\lim_{x\to 2^-} f(x)=\infty$

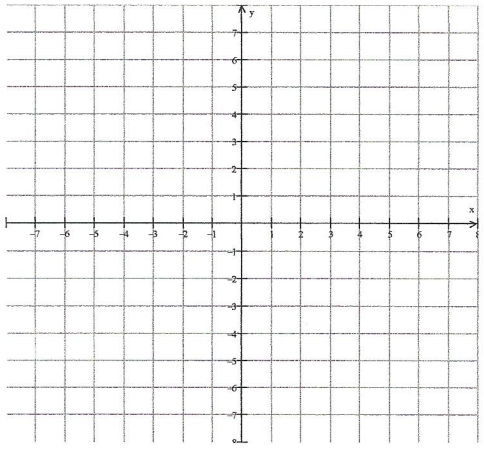
$\lim_{x\to 2^+} f(x)=-\infty$

$\lim_{x\to 3} f(x)=2$, but $f(3)=6$

$\lim_{x\to\infty} f(x)=5$

$\lim_{x\to-\infty} f(x)=5$

$\lim_{x\to 1} f(x)=0$, and $f(1)=0$



Limit Practice #2 _____ is takin' it to the LIMIT.

NO Calculator

Evaluate each limit. If a limit DOES NOT EXIST, tell why.

1. $\lim_{x\to 2}\left(2x^2+5x-7\right)$

2. $\lim_{x\to -\infty}\dfrac{5x}{x-2}$

3. $\lim_{x\to 1}\dfrac{x^2-4x+3}{x^2+6x-7}$

4. $\lim_{x\to 4^-}\dfrac{|x-4|}{x-4}$

5. $\lim_{x\to 0.5}[x]$

6. $\lim_{x\to 3}\dfrac{2x}{x-3}$

7. $\lim_{x\to \infty}\dfrac{5x^3-1}{3x+2}$

8. $\lim_{x\to -4} f(x)$ if $f(x)=\begin{cases}2x-10, & x<-4\\ 4x-2, & x>-4\end{cases}$

9. $\lim_{x\to 0}\dfrac{\sin(5x)}{2x}$

9. $\lim_{x\to 0}\dfrac{\sin(3x)}{12x-3x^2}$

10. $\lim_{x\to 0}\dfrac{\sin(5x)}{\sin(7x)}$

11. What are the three conditions that must be met for a function to be continuous at a point?

i.) _____

ii.) _____

iii.) _____

12. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

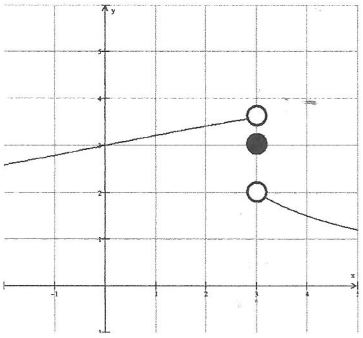
(a) $\lim_{x\to 0} f(x) =$ _____

(b) $\lim_{x\to 3^-} f(x) =$ _____

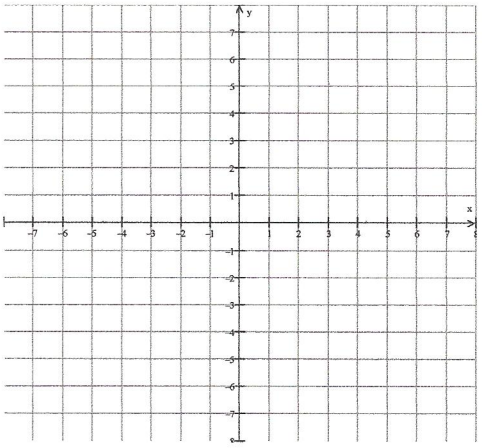
(c) $\lim_{x\to 3^+} f(x) =$ _____

(d) $\lim_{x\to 3} f(x) =$ _____

(e) $f(3) =$ _____



13. Graph $g(x) = \begin{cases} x-3, x<-2 \\ 4, x=-2 \\ x^2, x>-2 \end{cases}$.



14. Use the graph of f from question #12 and the graph of g from question #13 to find:

a. $\lim_{x\to 0}(f(x)+g(x)) =$ _____

b. $\lim_{x\to 0}\left(\frac{g(x)}{f(x)}\right) =$ _____

c. Write a limit using the functions $f(x)$ and $g(x)$ that does not exist and explain why it does not exist.

15. Sketch the graph of a function, h, that satisfies all of the given conditions.

$h(0) = 3$

$\lim_{x\to 0^-} h(x) = 4$

$\lim_{x\to 0^+} h(x) = 2$

$\lim_{x\to -\infty} h(x) = -\infty$

$\lim_{x\to \infty} h(x) = 3$

$\lim_{x\to 4^-} h(x) = -\infty$

$\lim_{x\to 4^+} h(x) = \infty$