LIMITS, CONTINUITY, IVT and SANDWICH THEOREM NAME ______________________

1. List the 3 conditions for continuity.

   ______________________ ______________________ ______________________

2. Use the Intermediate Value Theorem to show that there is a root of the equation $2x^3 + x^2 + 2 = 0$ in the interval (-2, -1).

3. If $2x - 1 \le x \le x^2$ for $0 < x < 3$, find $\lim_{x \to 1} f(x)$.

4. Prove that $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$.

5. Sketch the graph of a function, f, that satisfies all of the following conditions:

   $\lim_{x \to 0^+} f(x) = -2$ $\lim_{x \to 0^-} f(x) = 1$ $f(0) = -1$

   $\lim_{x \to 2^-} f(x) = \infty$ $\lim_{x \to 2^+} f(x) = -\infty$

6. Evaluate each limit:

   a. $\lim_{x \to 4} \frac{x-4}{x^2 - 3x - 4}$ b. $\lim_{x \to 8^-} \frac{|x-8|}{x-8}$

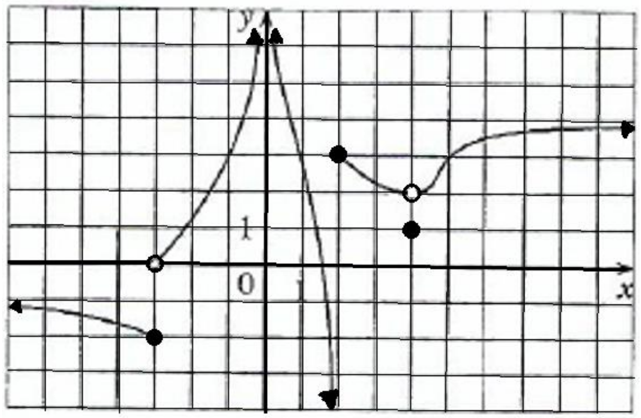
   c. $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x^2 - 4}$ d. $\lim_{x \to -6^+} \frac{x}{x+6}$

7. Given the function: $f(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ 3 - x, & 0 \le x < 3 \\ (x-3)^2, & x > 3 \end{cases}$, evaluate each limit:

$\lim_{x \to 0^+} f(x) =$ $\lim_{x \to 0^-} f(x) =$ $\lim_{x \to 0} f(x) =$

$\lim_{x \to 3^-} f(x) =$ $\lim_{x \to 3^+} f(x) =$ $\lim_{x \to 3} f(x) =$

8. Use interval notation to list intervals of continuity for the function shown in the graph.



9. Evaluate each limit:

a. $\lim\limits_{x\to-\infty} \dfrac{3}{x^2+1}$ b. $\lim\limits_{x\to\infty} \dfrac{3x^2+5}{7-2x^2}$ c. $\lim\limits_{x\to\infty} \dfrac{x^3}{2x^2-1}$

10. Find k so that the function is continuous on any interval.

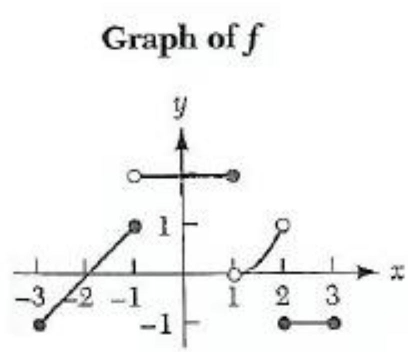
$$h(x)=\begin{cases} k\cos x, & 0\le x\le \pi \\ 12-\pi, & \pi < x \end{cases}$$

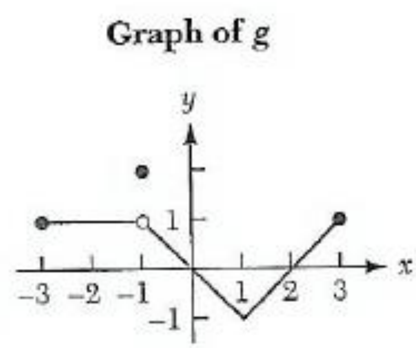
$$h(x)=\begin{cases} kx, & 0\le x\le 1 \\ 2kx+3, & 1< x\le 5 \end{cases}$$

$$h(x)=\begin{cases} 0.5x, & 0\le x< 1 \\ \sin(kx), & 1\le x\le 5 \end{cases}$$

$$h(x)=\begin{cases} \ln(kx+1), & 0\le x\le 2 \\ x+4, & 2< x\le 5 \end{cases}$$

11. Evaluate each limit. If a limit fails to exist, explain why.





(a) $\lim\limits_{x\to0}\left(f(x)+g(x)\right)$

(b) $\lim\limits_{x\to2}\left(f(x)+g(x)\right)$

(c) $\lim\limits_{x\to1}\left(f(x)\cdot g(x)\right)$

(d) $\lim\limits_{x\to2}\left(f(x)\cdot g(x)\right)$

(e) $\lim\limits_{x\to0}\dfrac{f(x)}{g(x)}$

(f) $\lim\limits_{x\to0}\left(f(x)\cdot\cos x\right)$

(g) $\lim\limits_{x\to-2} x^2 g(x)$