AB Calculus L'Hopital's Rule Name______________________________

Class Notes & Homework

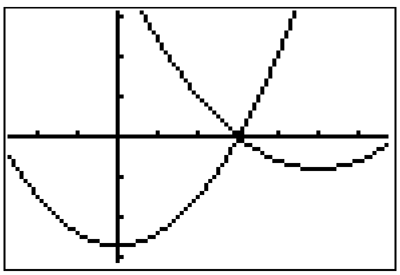
Consider $\lim\limits_{x\to 3}\dfrac{0.3x^2-2.7}{0.2x^2-2x+4.2}$. By direct substitution when $x\to 3$ yields $\lim\limits_{x\to 3}\dfrac{0.3x^2-2.7 \quad \to 0}{0.2x^2-2x+4.2 \quad \to 0}$ which implies there is a hole in the function at $x=3$, but recall it is the y-value of the hole that is the value of the limit.

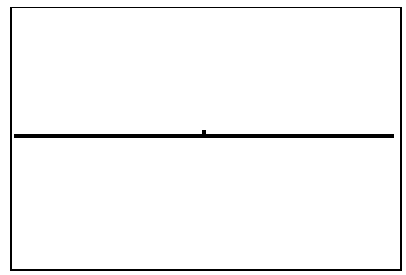
Use your graphing calculator to find estimate of the limit graphically or using the table. The limit is _______.

In other limit problems we would have used one of three analytical strategies to evaluate the limit:

1) factoring to reduce a hole,
2) multiplying both the numerator and the denominator by the conjugate factor to reduce a hole, or
3) simplifying complex fractions to reduce the hole.

There is a theorem that helps us find this limit analytically when these strategies are not effective, as in the example above. Consider the graphs of the numerator $Y_1=0.3x^2-2.7$ and denominator $Y_2=0.2x^2-2x+4.2$ in the window $[-2.7,6.7]x[-3.1,3.1]$. Graph these functions on your calculator.





Press TRACE, then press 3 and ENTER to place the tracing cursor on the point (3, 0) on the graph.
Press ZOOM, press 2 (Zoom In), press ENTER. After the graph re-draws, repeat the zooming in by pressing ENTER six more times. **Draw the final result in the blank window to the right of the graph above.**

What appears to be happening to the two curves with each successive Zoom In?____________________________
This phenomenon is called: _______________________________________

By zooming in we have created a picture of what appears to be two tangent lines to the curves at the point (3, 0). The two tangent lines approximate their two curves in this small neighborhood close to x=3. The ratio of Y1/Y2 is approximately equal to the ratio of the two linear functions pictured above.

Find the equation for each tangent line to the curve at the point (3, 0) using the calculator Tangent Line feature. Press 2nd PRGM for the DRAW menu. Choose #5: Tangent line. Your cursor should be on Y1 so press 3 and ENTER and wait for the tangent line equation to show up. Repeat but use the up or down arrow to jump to the Y2 curve. Record each tangent line below.

Factor the numerator & denominator…

$\dfrac{\textit{tangent line for } Y_1}{\textit{tangent line for } Y_2} =$ _______________________ What is the ratio of the slopes of the two tangent lines? ____

Now find $\dfrac{Y_1'(3)}{Y_2'(3)} =$ __________ What you notice about these ratios is summarized in **L'Hopital's Rule**.

L'Hopital's Rule: Functions $f(x)$ & $g(x)$ are differentiable where defined except possibly at $x=a$.

Given one of the following limits $\lim\limits_{x\to a}$, $\lim\limits_{x\to a+}$, $\lim\limits_{x\to a-}$, $\lim\limits_{x\to\infty}$, $\lim\limits_{x\to\infty+}$, $\lim\limits_{x\to\infty-}$ and

IF $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} g(x)=0$ OR $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} g(x)=\infty$, THEN $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}$

provided that $\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}=L$, a finite value, OR $\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}=\infty$.

To clarify, **L'Hopital's Rule** may be applied when a limit evaluates to an **indeterminate form**: $\dfrac{0}{0}$ & $\dfrac{\infty}{\infty}$

Five other **indeterminate forms** are: $\infty-\infty$, $0\cdot\infty$, 1^∞, 0^0, ∞^0. For these, you must re-express the limit as a ratio before applying L'Hopital's Rule.

Ex & Homework: Evaluate each limit using direct substitution to determine the limit or if L'Hopital's Rule applies.

1) $\lim\limits_{x\to 1}\dfrac{\ln(x)}{(x-1)}$	2) $\lim\limits_{x\to\infty} x^{-2}e^x$	3) $\lim\limits_{x\to\infty}\dfrac{\ln(x)}{\sqrt[3]{x}}$
4) $\lim\limits_{x\to\pi}\dfrac{\sin(x)}{(1-\cos(x))}$	5) $\lim\limits_{x\to 0}\dfrac{\sin(5x)}{(2x)}$	6) $\lim\limits_{x\to\infty}\dfrac{x^2+5}{(x+e^x)}$
7) $\lim\limits_{x\to 1}\dfrac{x^9-1}{x^5-1}$	8) $\lim\limits_{x\to 0}\dfrac{\sin(x)}{x^3}$	9) $\lim\limits_{x\to 0}\dfrac{e^x-1}{x^3}$
10) $\lim\limits_{x\to 0}\dfrac{5^x-3^x}{x}$	11) $\lim\limits_{x\to 0}\dfrac{\sin^{-1}(x)}{x}$	12) $\lim\limits_{x\to\infty}\dfrac{e^x}{x^3}$
13) $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos(x)}{1-\sin(x)}$	14) $\lim\limits_{x\to\infty} e^{-x}\ln(x)$	15) $\lim\limits_{x\to 0}\dfrac{1}{x}-\csc(x)$

16) $\lim_{x\to\infty} \frac{x}{\ln\left(1+2e^{x}\right)}$	17) $\lim_{x\to\pi} \left(x-\pi\right)\cot\left(x\right)$	18) $\lim_{x\to 0} \frac{\sin x - x}{x^{3}}$
19) $\lim_{x\to 0} \frac{\tan x - x}{x^{3}}$	20) $\lim_{x\to\infty} xe^{-x}$	21) $\lim_{x\to\infty} x^{3}e^{-x^{2}}$

22) $\lim_{x\to 0} x\csc\left(x\right)$

$A)\ -\infty \quad B)\ -1 \quad C)\ 0 \quad D)\ 1 \quad E)\ \infty$

23) $\lim_{x\to 0} \frac{1-\cos\left(x\right)}{2\sin^{2} x}$

$A)\ 0 \quad B)\ \frac{1}{8} \quad C)\ \frac{1}{4} \quad D)\ 1 \quad E)\ nonexistent$

24) $\lim_{x\to 0} \frac{1}{x}\ln\left(\frac{2+x}{2}\right)$

$A)\ e^{2} \quad B)\ 1 \quad C)\ \frac{1}{2} \quad D)\ 0 \quad E)\ nonexistent$

25) If $f'\left(x\right)=\cos\left(x\right)$ & $g'\left(x\right)=1$ for all x and if $f\left(0\right)=g\left(0\right)=0$, then find $\lim_{x\to 0} \frac{f\left(x\right)}{g\left(x\right)}$

$A)\ \frac{\pi}{2} \quad B)\ 1 \quad C)\ 0 \quad D)\ -1 \quad E)\ nonexistent$