KEY. REVIEW (J)

<u>Fill in the Blanks</u> for the Big Topics in Chapter 5: The Definite Integral $\int_a^b f(t)dt$

- Estimating an integral using a Riemann sum:
  1. The <u>Left rule</u> uses the left endpoint of each subinterval.
  2. The <u>Right rule</u> uses the right endpoint of each subinterval.
  3. The <u>Midpoint rule</u> uses the midpoint of each subinterval.
  4. The <u>Trapezoid rule</u> uses the average from the left and right rules, i.e.

  Formula for Trapezoid estimate using Left and Right estimates: $Trap(n) =$

- If the graph of f is increasing on [a, b], then LHS $\le \int_a^b f(x)dx \le$ RHS
- If the graph of f is decreasing on [a, b], then RHS $\le \int_a^b f(x)dx \le$ LHS

- If the graph of f is concave up on [a, b], then MID $\le \int_a^b f(x)dx \le$ TRAP
- If the graph of f is concave down on [a, b], then TRAP $\le \int_a^b f(x)dx \le$ MID

- $F(b) - F(a) = \int_a^b F'(t)dt =$ total change of $F(t)$ between $t = a$ and $t = b$

- Average value of f from a to b = $\frac{1}{(b-a)} \int_a^b f(t)dt$

- If f is <u>even</u>, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

- If g is <u>odd</u>, then $\int_{-a}^{a} g(x)dx = 0$

* • $\int_a^b f(x)dx = -\int_b^a f(x)dx$ * Given $a \le x \le b$, $\int_b^a f(x)dx = -\int_a^b f(x)dx$

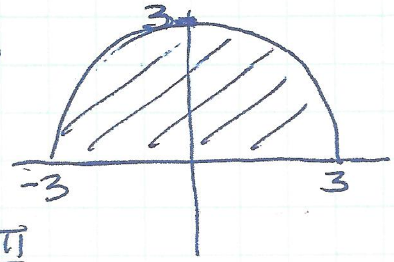
- $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$

- $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

- $\int_a^b cf(x)dx = c\int_a^b f(x)dx$

Ch5 REVIEW FRQ. (J)

① $y = \int_0^3 \sqrt{9-x^2}$

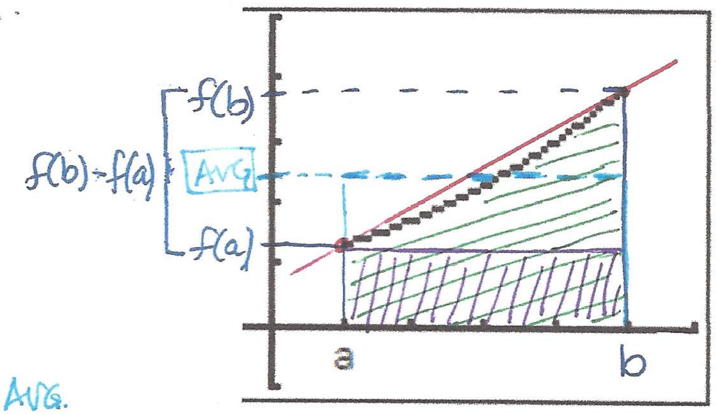


$= \frac{\pi(3)^2}{2} = \frac{9\pi}{2}$

② $\int_0^{12} f(x) = 18 \quad \int_0^9 f(x)\,dx = 10 \quad \therefore \int_9^{12} f(x)\,dx = \int_0^{12} f(x)\,dx - \int_0^9 f(x)\,dx$

$= 18 - 10 = 8$

3. Show the following on the graph:
   a) $f(b) - f(a)$ vertical length
   b) a line whose slope is $\boxed{\frac{f(b)-f(a)}{b-a}}$
   c) an area $F(b) - F(a)$ where $F' = f$
   d) $f(a)(b-a)$
   e) average value of $f = \boxed{\frac{1}{b-a}\int_a^b f(x)dx}$ = AVG.



④ $v(t) = 50(1.2)^t$ ft/sec

Average value on $[0, 10]$ sec.

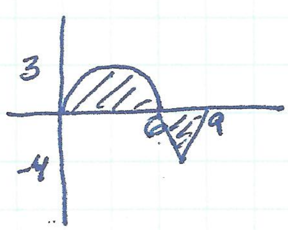
AVG Value of $v(t)$ $= \left(\frac{1}{10-0}\right)\int_0^{10} 50(1.2)^t$

$= 1423.785$ ft/sec

or $= 1423.786$

* Use calculator to evaluate. MATH:9 fnInt

⑤



A of semicircle + A of triangle

$\frac{1}{2}\pi(3)^2 + \frac{1}{2}(3)(-4)$

$\frac{9\pi}{2} - 6 = \frac{9\pi - 12}{2}$

⑥ d) AVG VALUE

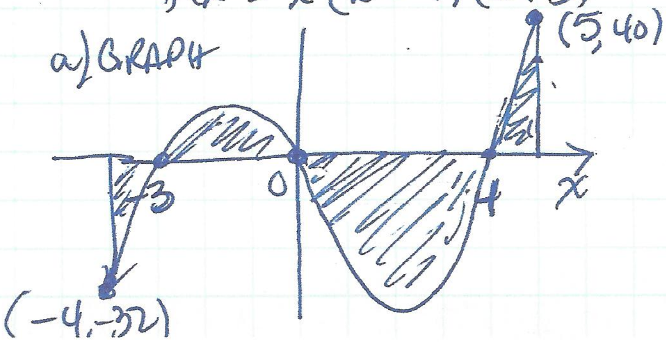
$\frac{1}{(5-{-4})}\int_{-4}^{5} f(x)dx = -2.75$

⑥ $f(x) = x^3 - x^2 - 12x$

$f(x) = x(x^2 - x - 12)$

$f(x) = x(x-4)(x+3)$

a) GRAPH



b) $\int_{-4}^{5} f(x)\,dx = \int_{-4}^{5}(x^3 - x^2 - 12x)\,dx = -24.75$

HOME SCREEN: MATH 9: fnInt

GRAPH 2nd CALC 7: $\int f(x)dx$

c) TOTAL AREA: (2 options for the answer)

i) $\int_{-4}^{-3} |f(x)|\,dx + \int_{-3}^{0} f(x)\,dx + \int_0^4 |f(x)|\,dx + \int_4^5 f(x)\,dx$

ii) $-\int_{-4}^{-3} f(x)\,dx + \int_{-3}^{0} f(x)\,dx - \int_0^4 f(x)\,dx + \int_4^5 f(x)\,dx$

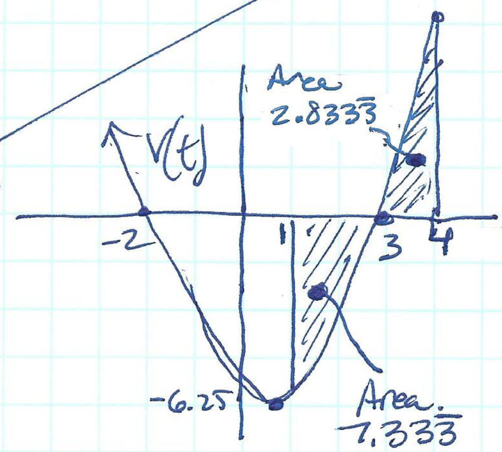
CH5
Review FRQ

J

⑦ $\int_0^5 f(x)\,dx = 5$ & $\int_0^2 f(x)\,dx = 3$ then $\int_2^5 f(x)\,dx = ?$

$$\int_0^5 f(x)\,dx - \int_0^2 f(x)\,dx = \int_2^5 f(x)\,dx$$
$$5 - 3 =$$
$$2 = \int_2^5 f(x)\,dx.$$

⑧ $v(t) = t^2 - t - 6$
$v(t) = (t-3)(t+2)$



a) Displacement $= \int_1^4 v(t)\,dt = -4.5$

b) Total Distance Traveled $= \int_1^4 |v(t)|\,dt = 10.1666$

⑨ MIDPOINT RULE $y = \frac{1}{x}$

x	1	$\frac{11}{10}$	1.2	$\frac{13}{10}$	1.4	$\frac{15}{10}$	1.6	$\frac{17}{10}$	1.8	$\frac{19}{10}$	2
y	1	$\frac{10}{11}$	$\frac{5}{6}$	$\frac{10}{13}$	$\frac{5}{7}$	$\frac{10}{15}$	$\frac{5}{8}$	$\frac{10}{17}$	$\frac{5}{9}$	$\frac{10}{19}$	$\frac{1}{2}$

$n=5$ $\frac{1}{2}\left(\frac{2-1}{5}\right) = \frac{1}{2}\left(\frac{1}{5}\right) = \frac{1}{10}$

$$\int_1^2 \frac{1}{x}\,dx = \left(\frac{1}{5}\right)\left[\frac{10}{11} + \frac{10}{13} + \frac{10}{15} + \frac{10}{17} + \frac{10}{19}\right] \approx 0.691907$$
$$\approx 0.691$$
OR ≈ 0.692

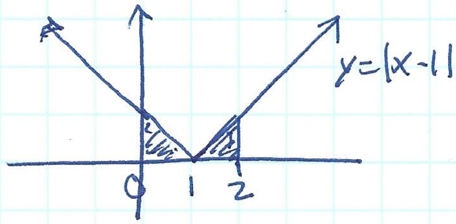
CH.5 REVIEW MC

p1 K

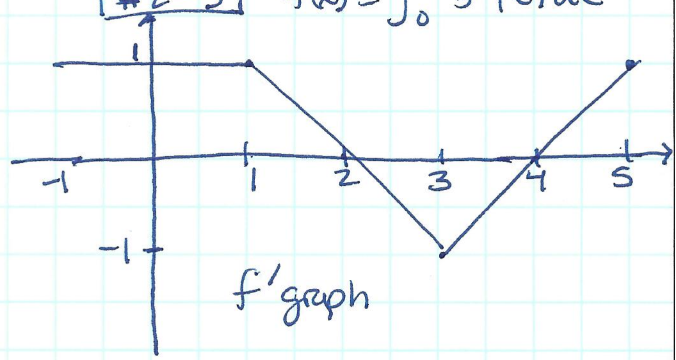
(1) $\int_0^2 |x-1|\,dx$

$= \frac{1}{2}+\frac{1}{2} = 1$

B



#2-5 $f(x)=\int_0^x f'(t)\,dt$



(2) I. $f(-1)=-1$

A

$f(-1)=\int_0^{-1} f(x)\,dx$

$= -\int_{-1}^{0} f(x)\,dx$

$= -1$ TRUE

II. $f(1) < f(3)$

$f(3)=\int_0^3 f(x)\,dx$

$= 1+\frac{1}{2}-\frac{1}{2} = 1$

$f(1)=\int_0 f(x)\,dx = 1$

$f(1)=f(3)$ FALSE

III. $f'(1) < f'(3)$

$f'(1)=1 < f'(3)=-1$

FALSE

(3) I. f is increasing on $(-1,2)$ only — FALSE also on $(4,5)$

B

II. f is inc on $(-1,2)$ & $(4,5)$ — TRUE ✓

III. f is decreasing on $(1,3)$ — FALSE only on $(2,4)$

(4) I f is concave up on $(-1,1)$ $f''(x)=0$ on $(-1,1)$ ∴ FALSE

b/c f' is constant f has no concavity on $(-1,1)$

E

II f is concave up on $(1,3)$ $f''(x)<0$ b/c f' is decreasing

∴ f ccdown on $(1,3)$ ∴ FALSE

III f is concave down on $(3,5)$ $f''(x)>0$ b/c f' is increasing

∴ f ccup on $(3,5)$ ∴ FALSE.

(5) I. f has rel. min @ $x=2$ f' changes signs ⊕ to ⊖ @ $x=2$ ∴ REL MAX. FALSE

E

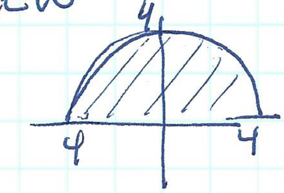
II. f has rel min @ $x=4$ f' changes signs ⊖ to ⊕ @ $x=4$ ∴ REL MIN. TRUE

III f has rel max @ $x=2$ TRUE

CH 5 MC REVIEW

p2 K

(6) $\int_{-4}^{4} \sqrt{16-x^2}\,dx$



Semicircle w/ radius 4.

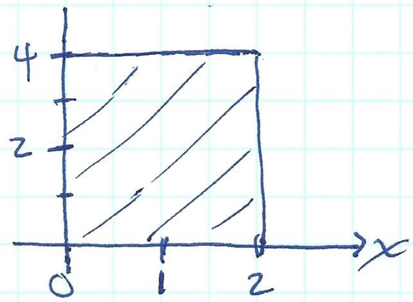
B

(7) $\int_{c}^{b} f(x)\,dx = \int_{a}^{b} f(x)\,dx - \int_{a}^{c} f(x)\,dx$

D

$a < c < b$

(8) $\int_{0}^{2} f(x)\,dx = (2)(4) = 8$



D

(9) $\int_{1}^{10} f(x)\,dx = 4 \qquad \int_{10}^{3} f(x)\,dx = 7$

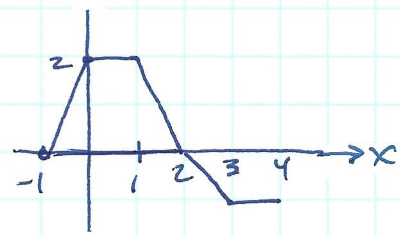
$\therefore \int_{3}^{10} f(x)\,dx = -7$

E

$\int_{1}^{3} f(x)\,dx = \int_{1}^{10} f(x)\,dx - \int_{3}^{10} f(x)\,dx =$

$= 4 - (-7)$

$= 11$



(10) $\int_{-1}^{4} f(x)\,dx$ = area of trap $(-1, 2)$ − area of trap $(2, 4)$

B

$= \left(\frac{1}{2}\right)(2)(1+3) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(2+1) = 4 - \frac{3}{2} = \frac{8}{2} - \frac{3}{2} = \frac{5}{2} = 2.5$

(11) $\int_{a}^{b} f(x)\,dx = F(b) - F(a)$

D

The Fundamental Theorem of Calculus.

(12) $v(t) = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ m/sec.

D

Total distance traveled on $[0, 4]$ $= \int_{0}^{4} |v(t)|\,dt = 80$

(13) The bug changes direction @ $\underline{t=6}$ b/c $v(t)$ changes signs + to −

C

(14) Total distance traveled $= \int_{0}^{8} |v(t)|\,dt =$ Area of TRAP + |Area of Triangle|

B

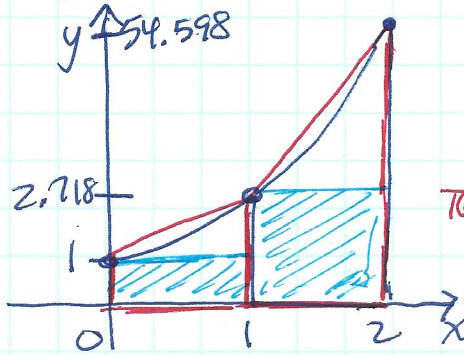
$= \frac{1}{2}(3)(2+6) + \frac{1}{2}(2)(1) = 12 + 1 = 13$

CH 5 MC REVIEW

0.3 K

(15) $\int_0^2 e^{x^2}\,dx$

D



TRAPEZOIDS

INSCRIBED RECTANGLES.

$\text{LHS} = (1)(1 + \overset{e^1}{2.718\ldots})$
$= 1 + e \approx 3.718$

$\text{TRAP} = (1)(\tfrac{1}{2})(1 + e^1 + e^1 + e^4)$
$= \tfrac{1}{2}(1 + 2e + e^4) \approx 30.517$

difference TRAP − LHS
$= 26.799$

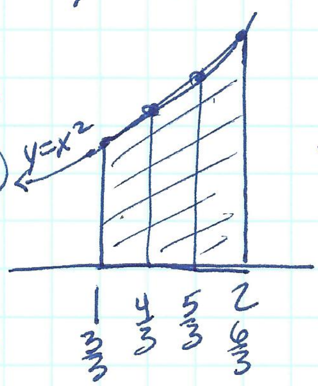
(*) (16) $v(t) = e^t$ Total distance $\int_0^2 e^t\,dt = e^t\Big|_0^2 = e^2 - e^0$
$= e^2 - 1$

A

(17) $y = \sqrt[3]{x+3}$ on $[-3,-2]$ AVG VALUE of $y = \dfrac{1}{-2+3}\displaystyle\int_{-3}^{-2} \sqrt[3]{x+3}\,dx$
$= 0.750$

B

(18)



D

$\text{TRAP} = \tfrac{1}{2}\left(\tfrac{1}{3}\right)\left(1 + \left(\tfrac{4}{3}\right)^2 + \left(\tfrac{4}{3}\right)^2 + \left(\tfrac{5}{3}\right)^2 + \left(\tfrac{5}{3}\right)^2 + (2)^2\right)$
$= \tfrac{1}{2}\left(\tfrac{1}{3}\right)\left(1 + 2\left(\tfrac{16}{9} + \tfrac{25}{9}\right) + 4\right) =$
$\tfrac{1}{6}\left(\tfrac{127}{9}\right) = \tfrac{127}{54} \approx 2.35185$
2.351 or 2.352

(19) TRAP. (note x-intervals are not consistent so be careful)

$\text{TRAP} = \tfrac{1}{2}(3)(10+30) + \tfrac{1}{2}(2)(30+40) + (\tfrac{1}{2})(1)(40+20)$

C

$= \tfrac{1}{2}(3(40) + 2(70) + 1(60)) = \tfrac{1}{2}(120+140+60) = 60+70+30$
$= 160.$

(20) $v(t) = 5t - t^2 + 100$ $t \in [0,4]$

D

AVG wind velocity $= \left(\dfrac{1}{4-0}\right)\displaystyle\int_0^4 v(t)\,dt = \tfrac{1}{4}(418.66\overline{6}) = 104.666$ $\dfrac{\text{miles}}{\text{hour}}$
104.667

(21) 7 weeks = 49 days. Total amount of pollutant dumped in river at end of 7 wks

$r(t) = \dfrac{\sqrt{t}}{180}$ $\dfrac{\text{tons}}{\text{day}}$

E

rate of pollutants dumped in river.

$= \displaystyle\int_0^{49} \frac{\sqrt{t}}{180}\,dt = 1.27037$ tons of pollutants.