

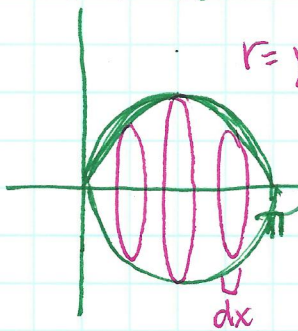
DAY 118 HW VOLUMES BY REVOLUTION (WASHERS)

packet p.15  
# 1-5

(DISKS)  
 $A = \pi r^2$   
 $V = \pi r^2 dx$

$A = \pi R^2 - \pi r^2$   
 $A = \pi (R^2 - r^2)$   
 $V = \pi (R^2 - r^2) dx \text{ (or } dy)$

①  $f(x) = \sqrt{\sin(x)}$  x-axis between  $(0, \pi)$  about x-axis.



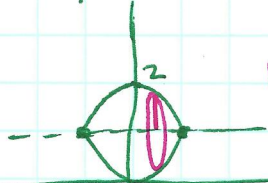
$r = y = \sqrt{\sin x}$   $A = \pi (\sin x)$

$V = \pi \int_0^\pi \sin x \, dx$

$V = \pi (-\cos x)|_0^\pi$

$V = \pi (1 - -1) = 2\pi$

②  $y = f(x) = 2 - x^2, y = 1$



$r = (2 - x^2) - 1 = 1 - x^2$

$A = \pi (1 - x^2)^2$

$A = \pi (1 - 2x^2 + x^4)$

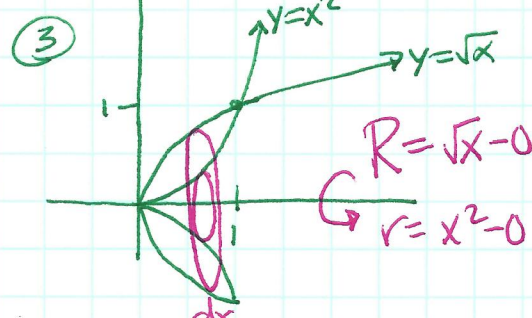
$V = \pi \int_{-1}^1 (1 - 2x^2 + x^4) \, dx$

$V = 2\pi \int_0^1 (1 - 2x^2 + x^4) \, dx$

$V = 2\pi (x - \frac{2}{3}x^3 + \frac{1}{5}x^5)|_0^1$

$V = 2\pi ((1 - \frac{2}{3} + \frac{1}{5}) - 0)$

$V = 2\pi (\frac{15 - 10 + 3}{15}) = \frac{16\pi}{15}$



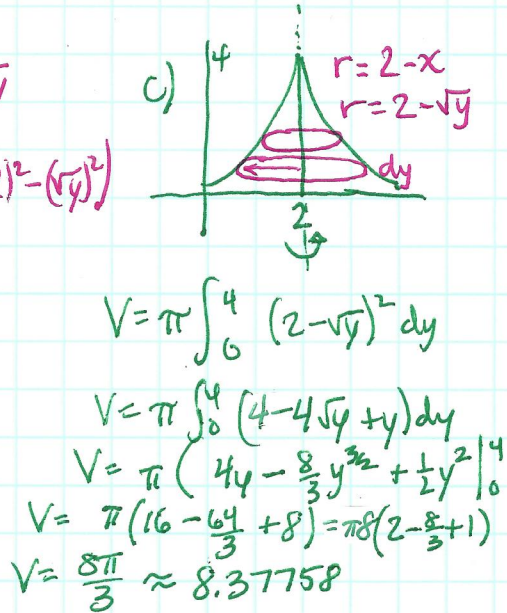
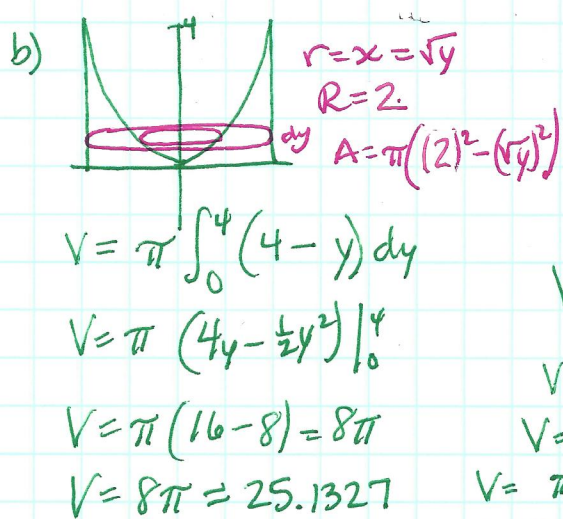
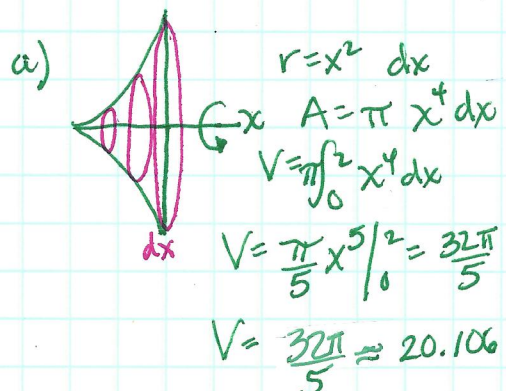
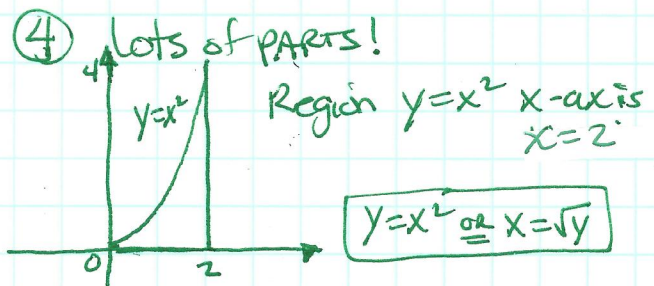
$R = \sqrt{x} - 0$   
 $r = x^2 - 0$

$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx$

$V = \pi \int_0^1 (x - x^4) \, dx$

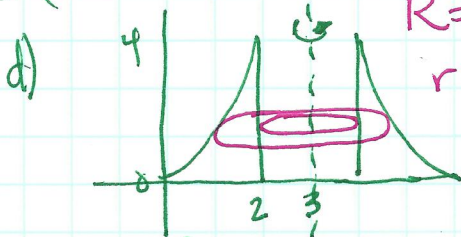
$V = \pi (\frac{1}{2}x^2 - \frac{1}{5}x^5)|_0^1$

$V = \pi (\frac{1}{2} - \frac{1}{5}) = \frac{3\pi}{10}$



DAY 18 HW

④ (continued)



$R = 3 - x = 3 - \sqrt{y}$   
 $r = 3 - 2 = 1$

$$V = \pi \int_0^4 (3 - \sqrt{y})^2 - (1)^2 dy$$

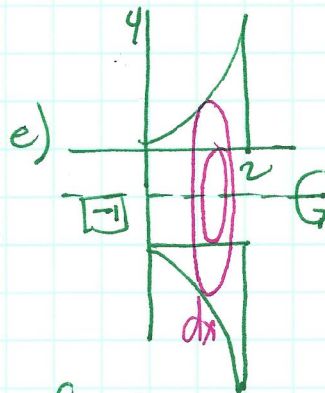
$$V = \pi \int_0^4 (9 - 6\sqrt{y} + y - 1) dy$$

$$V = \pi \int_0^4 (8 - 6\sqrt{y} + y) dy$$

$$V = \pi \left( 8y - 4y^{3/2} + \frac{1}{2}y^2 \right) \Big|_0^4$$

$$V = \pi ((32 - 32 + 8) - 0)$$

$$V = 8\pi \approx 25.13274$$



$R = y - 1 = x^2 + 1$   
 $r = 2 - 1 = 1$

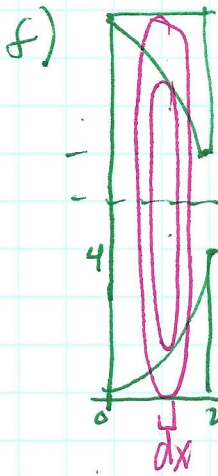
$$V = \pi \int_0^2 (x^2 + 1)^2 - (1)^2 dx$$

$$V = \pi \int_0^2 (x^4 + 2x^2 + 1 - 1) dx$$

$$V = \pi \left( \frac{1}{5}x^5 + \frac{2}{3}x^3 \right) \Big|_0^2 = \left( \frac{32}{5} + \frac{16}{3} \right) \pi$$

$$V = \pi 16 \left( \frac{2}{5} + \frac{1}{3} \right) = 16\pi \left( \frac{6+5}{15} \right) = \frac{176\pi}{15}$$

$$V = \frac{176\pi}{15} \approx 36.86635$$



$r = 5 - y = 5 - x^2$

$R = 5 - 0 = 5$

$$V = \pi \int_0^2 5^2 - (5 - x^2)^2 dx$$

$$V = \pi \int_0^2 25 - (25 - 10x^2 + x^4) dx$$

$$V = \pi \int_0^2 (10x^2 - x^4) dx$$

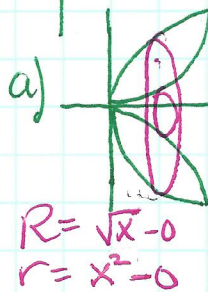
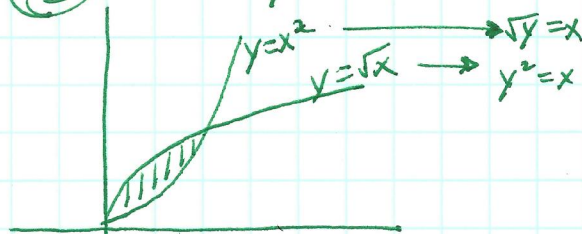
$$V = \pi \left( \frac{10}{3}x^3 - \frac{1}{5}x^5 \right) \Big|_0^2$$

$$V = \pi \left( \frac{80}{3} - \frac{32}{5} \right) = 8\pi \left( \frac{10}{3} - \frac{4}{5} \right)$$

$$V = 8\pi \left( \frac{38}{15} \right) = \frac{304\pi}{15} \quad 8\pi \left( \frac{50-12}{15} \right)$$

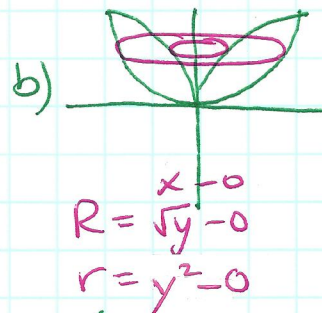
$$V = \frac{304\pi}{15} \approx 63.670$$

#5 LOTS of PARTS!



$R = \sqrt{x} - 0$   
 $r = x^2 - 0$

$$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 dx$$



$R = x - 0$   
 $r = y^2 - 0$

$$V = \pi \int_0^1 (\sqrt{y})^2 - (y^2)^2 dy$$

\* SAME B/C FUNCTIONS ARE INVERSES.

$$V = \pi \int_0^1 (x - x^4) dx$$

$$V = \pi \left( \frac{x^2}{2} - \frac{x^5}{5} \right) \Big|_0^1$$

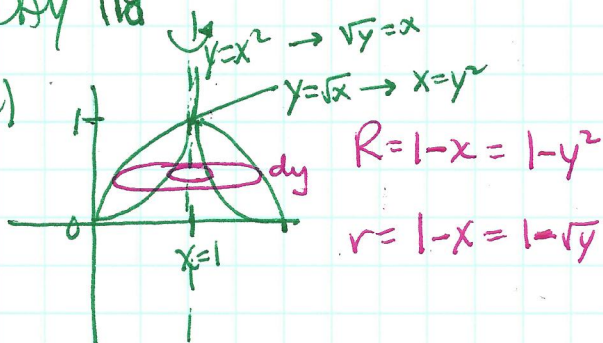
$$V = \pi \left( \frac{1}{2} - \frac{1}{5} \right)$$

$$V = \pi \frac{3}{10} \approx 0.942477$$

see part (a)

HW DAY 118

(5) c)



$$V = \pi \int_0^1 (1-y^2)^2 - (1-\sqrt{y})^2 dy$$

$$V = \pi \int_0^1 (1-2y^2+y^4) - (1-2\sqrt{y}+y) dy$$

$$V = \pi \int_0^1 (-2y^2+y^4+2\sqrt{y}-y) dy$$

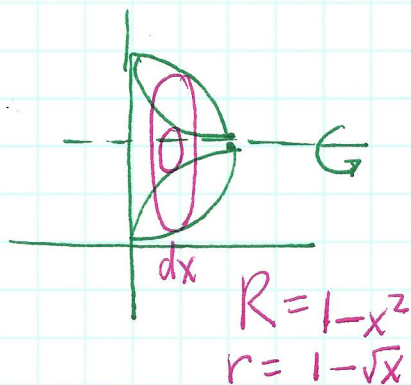
$$V = \pi \left( -\frac{2}{3}y^3 + \frac{1}{5}y^5 + \frac{4}{3}y^{\frac{3}{2}} - \frac{1}{2}y^2 \right) \Big|_0^1$$

$$V = \pi \left( -\frac{2}{3} + \frac{1}{5} + \frac{4}{3} - \frac{1}{2} \right)$$

$$V = \pi \left( \frac{2}{3} + \frac{1}{5} - \frac{1}{2} \right) = \pi \left( \frac{20+6-15}{30} \right)$$

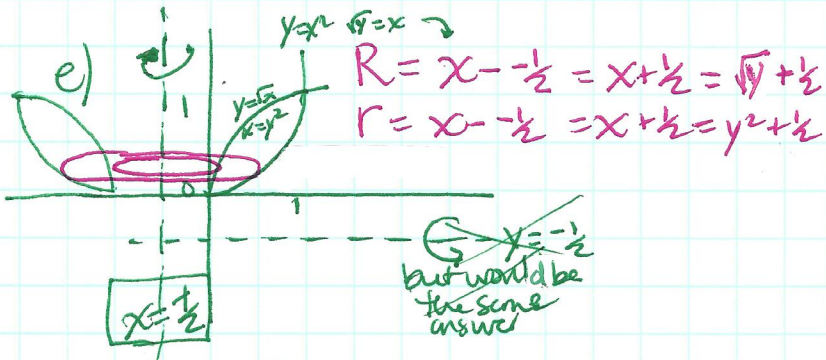
$$V = \frac{11\pi}{30} \approx 1.151917$$

d)



← Same as part (a)

$$V = \frac{11\pi}{30} \approx 1.151917$$



$$V = \pi \int_0^1 (\sqrt{y}+\frac{1}{2})^2 - (y^2+\frac{1}{2})^2 dy$$

$$V = \pi \int_0^1 (y + \sqrt{y} + \frac{1}{4}) - (y^4 + y^2 + \frac{1}{4}) dy$$

$$V = \pi \int_0^1 (y + \sqrt{y} - y^4 - y^2) dy$$

$$V = \pi \left( \frac{1}{2}y^2 + \frac{2}{3}y^{\frac{3}{2}} - \frac{1}{5}y^5 - \frac{1}{3}y^3 \right) \Big|_0^1$$

$$V = \pi \left( \left( \frac{1}{2} + \frac{2}{3} - \frac{1}{5} - \frac{1}{3} \right) - 0 \right)$$

$$V = \pi \left( \frac{1}{2} + \frac{1}{3} - \frac{1}{5} \right)$$

$$V = \pi \frac{15+10-6}{30} = \frac{19\pi}{30} \approx 1.989675$$