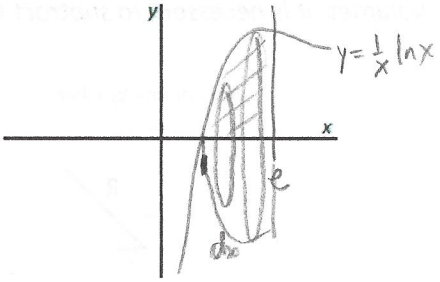
DAY ~~118~~ 117 HW Volumes of Revolution by Disks: AP FRQs Calculator Active #1-2 NON-Calculator #3

1. Let R be the region bounded by the graph of $y=\frac{1}{x}\ln(x)$, the x-axis, and the line $x=e$.

(a) Draw the region described.



(b) Find the area of the region R.

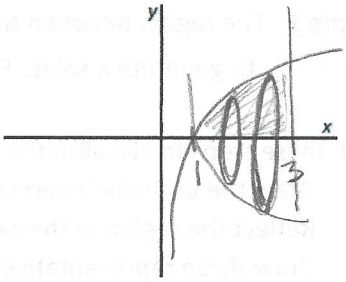
$A=\int_1^e \frac{1}{x}\ln x\,dx = \frac{1}{2}(\ln x)^2\Big|_1^e = \frac{1}{2}(1^2-0^2)=\frac{1}{2}$

(c) Find the volume of the solid formed by revolving the region R about the x-axis. dx-thickness

$V=\pi\int_1^e \left(\frac{\ln x}{x}\right)^2 dx = 0.160\pi \text{ or } 0.161\pi = 0.504$

2. Let R be the region enclosed by the graph of $y=\ln(x)$, the line $x=3$, and the x-axis.

(a) Graph the region described.



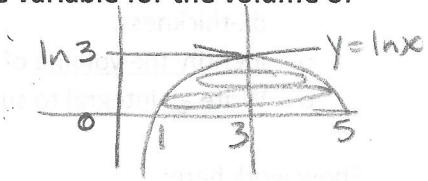
(b) Find the area of region R.

$A=\int_1^3 \ln x\,dx = 1.2958 = 1.295 = 1.296$

(c) Find the volume of the solid generated by revolving region R about the x-axis.

$V=\pi\int_1^3 (\ln x)^2\,dx = 3.23324$

(d) Set up, but do not evaluate, an integral expression in terms of a single variable for the volume of the solid generated by revolving region R about the line $x=3$.



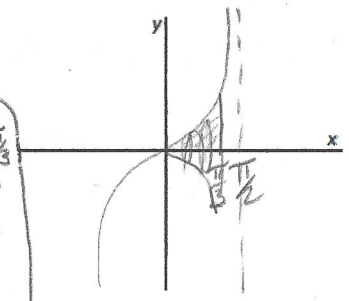
$y=\ln x$ $\quad r=3-x=3-e^y$

$x=e^y$

$V=\pi\int_0^{\ln 3}(3-e^y)^2\,dy$

3. (Non-Calculator) Let R be the region in the first quadrant that is enclosed by the graph of $y=\tan(x)$, the x-axis, and the line $x=\frac{\pi}{3}$.

a. Graph the region R.



b. Find the area of R. Answer: $(\ln 2)$

b) $\ln 2 \approx 0.693$

$\int_0^{\pi/3}\tan x\,dx = \ln|\sec x|\Big|_0^{\pi/3}$
$= \ln|\sec\frac{\pi}{3}| - \ln|\sec 0|$
$= \ln 2 - \ln 1$
$= \ln 2$
≈ 0.693147

$\cos\frac{\pi}{3}=\frac{1}{2}$

b. Find the volume of the solid formed by revolving the region R about the x-axis.

Answer: $\pi\left(\sqrt{3}-\frac{\pi}{3}\right)$

$\pi\int_0^{\pi/3}(\tan x)^2\,dx$ Use Pythagorean Trig Identity to re-express $\tan^2 x = \sec^2 x - 1$

$\pi\int_0^{\pi/3}(\sec^2 x-1)\,dx = \pi\left(\tan x - x\right)\Big|_0^{\pi/3}$

$\pi\left(\left(\tan\frac{\pi}{3}-\frac{\pi}{3}\right)-(\tan 0-0)\right) = \pi\left(\sqrt{3}-\frac{\pi}{3}\right)$