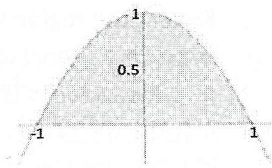
DAY 116 HW #1-8 below. Show all work on your own paper.

For each of the following, graph the region of the base, set up the integral to find the appropriate volume, then except for integrating #4 by hand, use your calculator to evaluate the integral. Answers are provided.

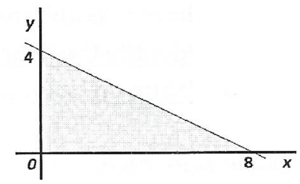
1. Find the volume of the solid with circular base of diameter 10 cm and whose cross-sections perpendicular to the x-axis are equilateral triangles. Answer: $\frac{500\sqrt{3}}{3} \approx 288.675$

2. The base of a solid is the region bounded by the graph of $y = 1 - x^2$ and the x-axis. For this solid, each cross section perpendicular to the x-axis is a rectangle with height three times the base. What is the volume of this solid? Answer: $\frac{48}{15} = \frac{16}{5} \approx 3.2$



3. The base of a solid is the region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

Answer: $\frac{16\pi}{3} \approx 16.755$



4. The base of a solid is bounded by $y = \cos(x)$, the x-axis, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Cross sections perpendicular to the x-axis are squares. Find the volume. **Calculate by hand using the fact that** $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$. Answer: $\frac{\pi}{2}$

5. The base of a solid is bounded by $y = 2 - x$, the x-axis, and the y-axis. Cross sections that are perpendicular to the x-axis are isosceles right triangles with the right angle on the x-axis. That is, the legs are perpendicular to the x-axis. Find the volume. Answer: $\frac{4}{3} \approx 1.333$

6. The base of a solid is bounded by the semi-circle $y = \sqrt{4 - x^2}$ and the x-axis. Cross sections that are perpendicular to the x-axis are squares. Find the volume. Answer: $\frac{32}{3} \approx 10.666$

7. The base of a solid is bounded by $y = \sqrt{16 - x^2}$ and the x-axis. Cross sections that are ***perpendicular to the Y-axis*** are equilateral triangles. Find the volume. Answer: $\frac{128\sqrt{3}}{3} \approx 73.901$ (Oops... if perpendicular to the x-axis then Answer: $\frac{64\sqrt{3}}{3} \approx 36.950$ = incorrect answer)

8. The base of a solid is bounded by $y = 2 - \frac{1}{2}x$, the x-axis, and the y-axis. Cross sections that are ***perpendicular to the Y-axis*** are isosceles right triangles with the hypotenuse in the xy-plane. Find the volume.

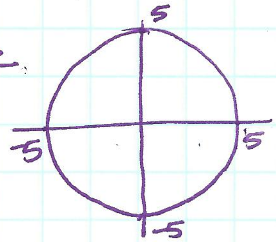
Answer: $\frac{8}{3} \approx 2.666$ (Oops... if perpendicular to the x-axis then Answer: $\frac{4}{3} \approx 1.333$ = incorrect answer)

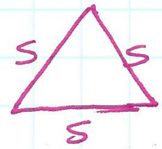
DAY 116 Volumes. Packet p.5 #1-8

① Circular base

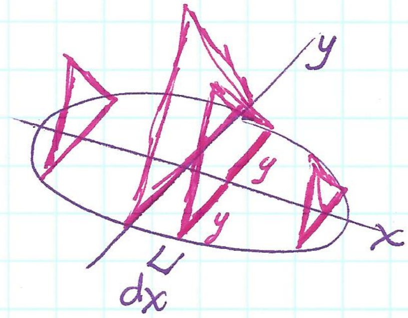
$x^2+y^2=25$

$y=\pm\sqrt{25-x^2}$





$A=\frac{\sqrt{3}}{4}s^2$



$\rightarrow 2y=s \rightarrow s=2\sqrt{25-x^2}$

Area of one △ : $A=\frac{\sqrt{3}}{4}\left(2\sqrt{25-x^2}\right)^2=\sqrt{3}\,(25-x^2)$

w/ dx-thickness

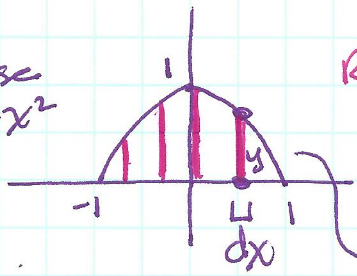
Volume :
$$\boxed{V=\int_{-5}^{5}\sqrt{3}\,(25-x^2)\,dx \quad \text{OR} \quad V=2\sqrt{3}\int_0^5 (25-x^2)\,dx}=2\sqrt{3}\left(25x-\tfrac{1}{3}x^3\right|_0^5$$

$$=2\sqrt{3}\left[\left(125-\tfrac{125}{3}\right)-(0)\right]=\frac{2\sqrt{3}}{3}(375-125)$$

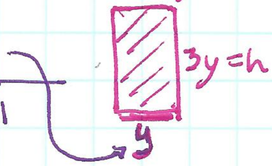
$$=\frac{2\sqrt{3}}{3}\cdot 250=\boxed{\frac{500\sqrt{3}}{3}}$$

288.675

② Base $y=1-x^2$



Rectangle



Area of one Rectangle $=(b)(h)$
$=(y)(3y)$
$=(1-x^2)(3(1-x^2))$
$=3(1-x^2)^2$

w/dx thickness

$$\boxed{\text{Volume}=\int_{-1}^{1}3(1-x^2)^2\,dx \overset{\text{OR}}{=} 2\int_0^1 3(1-x^2)^2\,dx}$$

X not a u-sub

must expand

$$V=6\int_0^1(1-x^2)^2\,dx=6\int_0^1 1-2x^2+x^4\,dx=6\left(x-\tfrac{2}{3}x^3+\tfrac{1}{5}x^5\right|_0^1$$

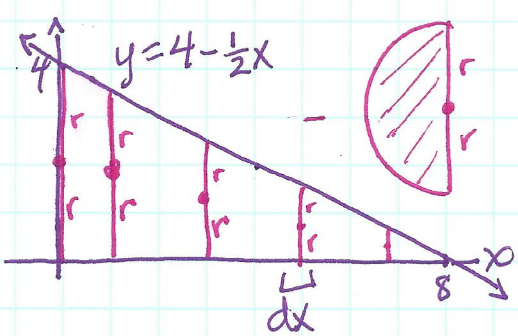
$$=6\left[\left(1-\tfrac{2}{3}+\tfrac{1}{5}\right)-(0)\right]=6\left[\tfrac{15}{15}-\tfrac{10}{15}+\tfrac{3}{15}\right]=\boxed{\frac{48}{15}}$$

3.2

③ Base

$x+2y=8$

$y=-\frac{1}{2}x+4$



$\frac{1}{2}\pi r^2=A$

$r=\frac{1}{2}y=\frac{1}{2}\left(4-\frac{1}{2}x\right)=2-\frac{1}{4}x=r$

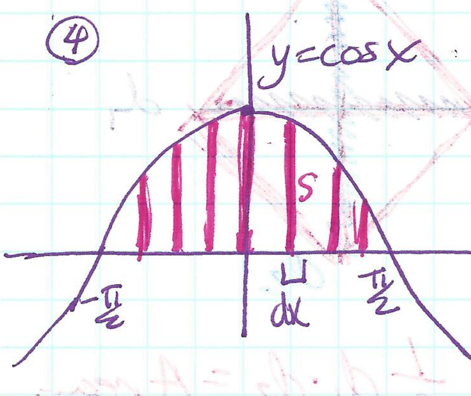
Area of one semicircle $=\frac{\pi}{2}\left(2-\frac{1}{4}x\right)^2$

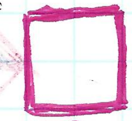
dx-thickness

$$\therefore \boxed{\text{Volume}=\frac{\pi}{2}\int_0^8\left(2-\tfrac{1}{4}x\right)^2dx}\approx\boxed{16.755}$$

DAY 116 packet p.5 #4-8 continued

④





$A=(s)^2 = (\cos x)^2$

$s=\cos x$

dx thickness

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$V=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x \, dx$

OR

$V=2\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

$V= 2 \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(1-\cos(2x)\right) dx$

$V=\int_0^{\frac{\pi}{2}} 1-\cos(2x)\, dx$

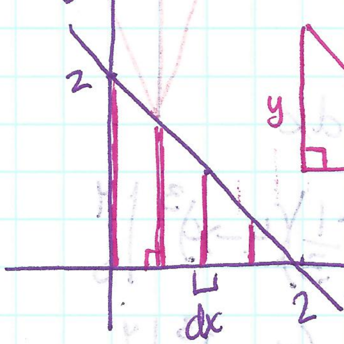
$= x - \frac{1}{2}\sin(2x) \Big|_0^{\frac{\pi}{2}}$

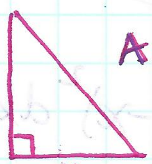
$=\left(\frac{\pi}{2}-\frac{1}{2}\sin\pi\right)-(0-0) = \boxed{\frac{\pi}{2}}$

$u=2x$
$\frac{1}{2}du=dx$

$\int \cos(2x)\, dx$
$=\frac{1}{2}\sin(2x)+c$

⑤ $y=2-x$





$A=\frac{1}{2}y^2$

$A=\frac{(2-x)^2}{2}$

$A=\frac{1}{2}(2-x)^2$

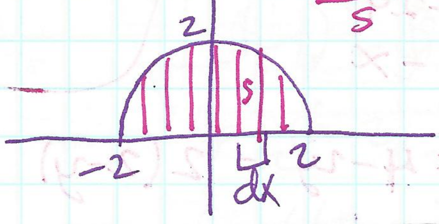
$\text{Volume} = \frac{1}{2}\int_0^2 (2-x)^2\, dx$

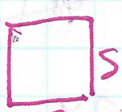
$= -\frac{1}{6}(2-x)^3 \Big|_0^2$

$u=2-x$
$du=-dx$

$= \frac{-1}{6}\left[(0)-(2)^3\right] = \frac{8}{6} = \boxed{\frac{4}{3}}$

⑥ $y=\sqrt{4-x^2}=s$





$A=s^2$

$A=(4-x^2)$

$\text{Volume}=\int_{-2}^{2}(4-x^2)\, dx$

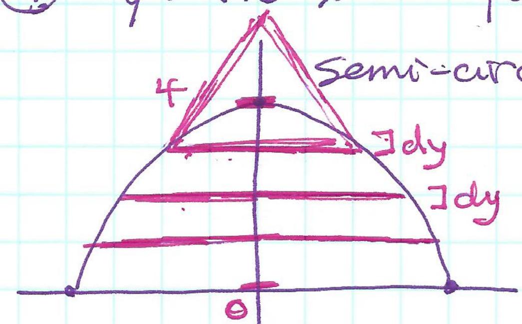
$V = 2\int_0^2 (4-x^2)\, dx$

$V = 2\left(4x-\frac{1}{3}x^3\Big|_0^2\right.$

$V = 2\left[\left(8-\frac{8}{3}\right)-0\right] = \boxed{\frac{32}{3}}$

DAY 116 packet p.5 # 7-8

(7) $y=\sqrt{16-x^2}$ perpendicular to the y-axis not x-axis!



semi-circle as base dy-thickness

equilateral triangles w/ side $= s = 2x$

$y^2=16-x^2$

$x^2=16-y^2$

$x=\sqrt{16-y^2}$

$s=2\sqrt{16-y^2}$

y-limits [0,4]

$$A=\frac{\sqrt{3}}{4}s^2=\frac{\sqrt{3}}{4}\left(2\sqrt{16-y^2}\right)^2=\sqrt{3}\,(16-y^2)$$

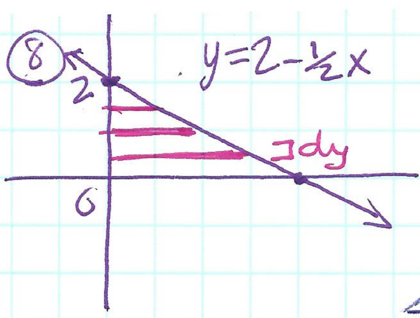
$$\text{Volume}=\sqrt{3}\int_0^4(16-y^2)\,dy$$

$$=\sqrt{3}\left(16y-\tfrac{1}{3}y^3\right|_0^4 = \sqrt{3}\left(16y-\tfrac{1}{3}y^3\right|_0^4$$

$$\sqrt{3}\left(64-\frac{64}{3}\right)$$

$$64\sqrt{3}\left(1-\tfrac{1}{3}\right)=\boxed{\frac{128\sqrt{3}}{3}}$$

(8)



perpendicular to the y-axis not x-axis!

dy-thickness

Isosceles Right Triangles with "diagonal" hypotenuse in the base

diagonal $d=x=(4-2y)=d$

$2y=4-x$

$x=4-2y$

y-limits [0,2]

$$A=\frac{1}{4}(d)^2=\frac{1}{4}(4-2y)^2=\frac{1}{4}\left(2(2-y)\right)^2=(2-y)^2$$

$$\text{Volume}=\int_0^2(2-y)^2\,dy=-\int_2^0u^2\,du=\int_0^2u^2\,du$$

$u=2-y$

$du=-dy$

$-du=dy$

$$=\frac{1}{3}u^3\Big|_0^2$$

$$=\frac{1}{3}(8-0)$$

$$=\boxed{\frac{8}{3}}$$