DAY 113 p.360 #57-69, 79-82, 104-106

(57) $\int_0^{\pi} \cos(x+\pi)\,dx = -\sin(x+\pi)\Big|_0^{\pi} = -\sin(2\pi) + \sin(\pi) = \boxed{0}$

$u = x+\pi$
$du = dx$ ∴ no usub needed

(58) $\int_0^{1/2} \cos(\pi x)\,dx \rightarrow \int_0^{\pi/2} \cos u\,du = -\sin u\Big|_0^{\pi/2} = -\sin\left(\frac{\pi}{2}\right) + \sin(0)$
$= -1 + 0$
$= \boxed{-1}$

$u = \pi x$
$du = \pi\,dx$
$\frac{1}{\pi}du = dx$

$u = \pi x$ $\quad x = \frac{1}{2} \rightarrow u = \frac{\pi}{2}$
$x = 0 \rightarrow u = 0$

(59) $\int_0^{\pi/2} e^{-\cos\theta} \sin\theta\,d\theta = \int_{-1}^{0} e^u\,du = e^u\Big|_{-1}^{0} = e^0 - e^{-1} = \boxed{1 - \frac{1}{e}}$

$u = -\cos\theta \longrightarrow x = \frac{\pi}{2} \rightarrow u = 0$
$du = \sin\theta\,d\theta \quad x = 0 \rightarrow u = -1$

(60) $\int_1^2 2xe^{x^2}\,dx = \int_1^4 e^u\,du = e^u\Big|_1^4 = \boxed{e^4 - e^1 = e(e^3-1)}$

$u = x^2 \longrightarrow x = 2 \rightarrow u = 4$
$x = 1 \rightarrow u = 1$
$du = 2x\,dx$

(61) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_1^2 e^u\,du = 2e^u\Big|_1^2 = 2(e^2 - e^1) = \boxed{2e(e-1)}$

$u = \sqrt{x} \longrightarrow x = 4 \rightarrow u = 2$
$du = \frac{1}{2\sqrt{x}}dx \quad x = 1 \rightarrow u = 1$
$2du = \frac{dx}{\sqrt{x}}$

(62) $\int_{-1}^{e-2} \frac{1}{t+2}\,dt = \ln|t+2|\Big|_{-1}^{e-2} = \ln|e| - \ln|1| = 1 - 0 = \boxed{1}$

$u = t+2$
$du = dt$ no usub necessary

(63) $\int_1^4 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int_1^2 \cos u\,du = 2(\sin(u)\Big|_1^2 = 2(\sin(2) - \sin(1))$
$= \boxed{2(\sin 2 - \sin 1)}$

$u = \sqrt{x} \longrightarrow x = 4 \rightarrow u = 2$
$x = 1 \rightarrow u = 1$
$du = \frac{1}{2\sqrt{x}}dx$
$2du = \frac{dx}{\sqrt{x}}$

(64) $\int_0^2 \frac{x}{(1+x^2)^2}\,dx = \int_3^5 \frac{1}{u^2}\,du = -\frac{1}{u}\Big|_3^5 = -\frac{1}{5} - -\frac{1}{3} = \frac{1}{3} - \frac{1}{5} = \boxed{\frac{2}{15}}$

$u = 1+x^2 \longrightarrow x = 2 \rightarrow u = 3$
$du = 2x\,dx \quad x = 0 \rightarrow u = 5$
$\frac{1}{2}du = x\,dx$

(65) $\int_{-1}^{3} (x^3 + 5x)\,dx = \frac{1}{4}x^4 + \frac{5}{2}x^2\Big|_{-1}^{3} = \frac{x^2}{4}(x^2+10)\Big|_{-1}^{3} = \left(\frac{9}{4}\right)(19) - \left(\frac{1}{4}\right)(11) =$
$\frac{1}{4}(171 - 11) = \frac{160}{4} = \boxed{40}$

$\boxed{\text{DAY 113}}$ p.360 #66–69, 79, 104–106

Continued

(66) $\int_{-1}^{1} \frac{1}{1+y^2}\,dy = \arctan(y)\Big|_{-1}^{1} = \arctan(+1) - \arctan(-1)$
$= \left(\frac{\pi}{4}\right) - \left(-\frac{\pi}{4}\right) = \boxed{\frac{\pi}{2}}$

Yes you should know these arctan values. ☺

(67) $\int_1^3 \frac{1}{x}\,dx = \ln|x|\Big|_1^3 = \ln|3| - \ln|1| = \boxed{\ln|3|}$

(69) $\int_{-1}^{2} \sqrt{x+2}\,dx = \frac{2}{3}(x+2)^{3/2}\Big|_{-1}^{2} = \frac{2}{3}\left(4^{3/2} - 1^{3/2}\right) = \frac{2}{3}(8-1) = \boxed{\frac{14}{3}}$

(68) $\int_1^3 \frac{dt}{(t+7)^2} = \frac{-1}{(t+7)}\Big|_1^3 = \frac{-1}{10} - \frac{-1}{8} = \frac{-1}{10} + \frac{1}{8} = \frac{-4+5}{40} = \boxed{\frac{1}{40}}$

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(79) $\int_0^{\pi/3} 3\sin^2(3x)\,dx = \int_0^{\pi} \sin^2(u)\,du$ ✓

$u = 3x \longrightarrow x = \frac{\pi}{3} \rightarrow u = \pi$
$du = 3dx \qquad x = 0 \rightarrow u = 0$

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(104) $\int_1^5 \frac{3x\,dx}{\sqrt{5x^2+7}}$

$w = 5x^2+7 \rightarrow x=5 \;\therefore w = 132$
$dw = 10x\,dx \searrow x = 1 \;\therefore w = 12$
$\frac{1}{10}dw = x\,dx$

$\frac{3}{10}\int_{12}^{132} \frac{dw}{\sqrt{w}} = \frac{3}{10}\int_{12}^{132} w^{-1/2}\,dw$

$\boxed{k = \frac{3}{10} \quad n = -\frac{1}{2} \quad w_0 = 12 \quad w_1 = 132}$

(105) $\int_0^5 \frac{2^x\,dx}{2^x+3}$

$w = 2^x+3 \rightarrow x=5,\; w = 35$
$dw = \ln 2\,(2^x)\,dx \searrow x = 0,\; w = 4$
$\frac{1}{\ln 2}dw = 2^x\,dx$

$\frac{1}{\ln 2}\int_4^{35} \frac{dw}{w}$

$\boxed{k = \frac{1}{\ln 2} \quad n = -1 \quad w_0 = 4 \quad w_1 = 35}$

(106) $\int_{\pi/12}^{\pi/4} \sin^7(2x)\cos(2x)\,dx$

$w = \sin 2x \rightarrow x = \frac{\pi}{4} \quad u = \sin\frac{\pi}{2} = 1$
$\rightarrow x = \frac{\pi}{12} \quad u = \sin\frac{\pi}{6} = \frac{1}{2}$
$dw = 2\cos(2x)dx$
$\frac{1}{2}dw = \cos(2x)dx$

$\frac{1}{2}\int_{1/2}^{1} w^7\,dw$

$\boxed{k = \frac{1}{2} \quad n = 7 \quad w_0 = \frac{1}{2} \quad w_1 = 1}$