CH 6 HW DAY 103 §6.4

p. 343 #35–38, 45, T/F 48–53 p. 347 #73–76 ✓

(35) $\frac{d}{dx}\int_0^{x^2} \ln(1+t^2)\,dt = \ln(1+x^4)(2x) = \boxed{2x \cdot \ln(1+x^4)}$

(36) $\frac{d}{dx}\int_1^{\sin t} \cos(x^2)\,dx = \boxed{\cos(\sin^2 t)\cdot \cos t}$

(37) $\frac{d}{dt}\int_{2t}^{4} \sin(\sqrt{x})\,dx = \frac{d}{dx}\int_4^{2t} -\sin(\sqrt{x})\,dx = \boxed{-2\sin\sqrt{2t}}$

(38) $\frac{d}{dx}\int_{-x^2}^{x^2} e^{t^2}\,dt = \frac{d}{dx}\int_{-x^2}^{0} e^{t^2}\,dt + \frac{d}{dx}\int_0^{x^2} e^{t^2}\,dt$

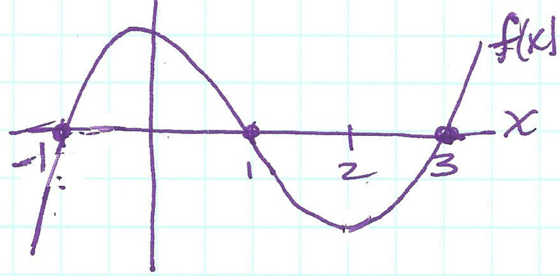
$= \frac{d}{dx}\int_0^{-x^2} - e^{t^2}\,dt + \frac{d}{dx}\int_0^{x^2} e^{t^2}\,dt$

$= -e^{x^4}(-2x) + e^{x^4}(2x)$

$= 2xe^{x^4} + 2xe^{x^4} = \boxed{4xe^{x^4}}$

(45) $\boxed{F(x) = \int_0^x f(t)\,dt \text{ has a local minimum @ } x=2}$

This statement is incorrect because...



$F' = f$ changes signs from ⊖ to ⊕ at $x=3$ $\therefore$ $F(x)$ has a local minimum at $x=3$ not at $x=2$.

$F(x)$ has a point of inflection at $x=2$ b/c $F' = f$ changes from decreasing to increasing which means $F'' = f'$ changes sign from ⊖ to ⊕ and $F(x)$ changes concavity

T/F

(48) every continuous function has an antiderivative TRUE

(49) $\int_0^x \sin(t^2)\,dt$ is an antiderivative of $\sin(x^2)$ TRUE

(50) $F(x) = \int_0^x f(t)\,dt$, then $F(5) - F(3) = \int_3^5 f(t)\,dt$ TRUE

(51) $F(x) = \int_0^x f(t)\,dt$, then $F(x)$ must be increasing (FALSE) only when $f(t) > 0$.

(52) $F(x) = \int_0^x f(t)\,dt$ & $G(x) = \int_2^x f(t)\,dt$ then $F(x) = G(x) + C$ (FALSE) → but $\boxed{F(x) + C = G(x)}$

(53) If $F(x) = \int_0^x f(t)\,dt$ and $G(x) = \int_0^x g(t)\,dt$ then $F(x) + G(x) = \int_0^x f(t) + g(t)\,dt$ TRUE.

HW $\boxed{\text{DAY 103}}$ continued p. 347 #73–76

(73) $\frac{d}{dx}\int_2^{x^3} \sin(t^2)\,dt = \sin(x^6)\cdot(3x^2) = \boxed{3x^2\sin(x^6)}$

(74) $\frac{d}{dx}\int_{\cos x}^{3} e^{t^2}\,dt = \frac{d}{dx}\int_3^{\cos x} -e^{t^2}\,dt = \left(+e^{\cos^2 x}\right)\cdot(\sin x)$

$= \boxed{\sin x\, e^{\cos^2 x}}$

(75) $\underbrace{\frac{d}{dx}\int_{-x}^{x} e^{-t^4}\,dt}_{}$ $= \frac{d}{dx}\int_{-x}^{0} e^{-t^4}\,dt + \frac{d}{dx}\int_0^{x} e^{-t^4}\,dt$

even function

so

$\frac{d}{dx}\, 2\int_0^{x} e^{-t^2}\,dt$

$= \boxed{2e^{-x^2}}$

$= \frac{d}{dx}\int_0^{-x} -e^{-t^4}\,dt + \frac{d}{dx}\int_0^{x} e^{-t^4}\,dt$

$= -e^{-x^4}(-1) + e^{-x^4}$

$= e^{-x^4} + e^{-x^4}$

$= \boxed{2e^{-x^4}}$

(76) $\frac{d}{dt}\int_{e^t}^{t^3} \sqrt{1+x^2}\,dx = \frac{d}{dx}\int_{e^t}^{0} \sqrt{1+x^2}\,dx + \frac{d}{dx}\int_0^{t^3} \sqrt{1+x^2}\,dx$

$= \frac{d}{dx}\int_0^{e^t} -\sqrt{1+x^2}\,dx + \frac{d}{dx}\int_0^{t^3} \sqrt{1+x^2}\,dx$

$= -\sqrt{1+e^{2t}}\,(e^t) + \sqrt{1+t^6}\cdot(3t^2)$

$= \boxed{(3t^2)\sqrt{1+t^6} - e^t\sqrt{1+e^{2t}}}$