Ch 5.4 PROPERTIES OF INTEGRALS & THEOREMS. DAY 87 (D)

Pg 305-307 #1-6, 22, 23 / 24, 26-28 / 29, 31, 37

#1-6) $\int_a^b f(x)\,dx = 8$ $\int_a^b (f(x))^2\,dx = 12$ $\int_a^b g(x)\,dx = 2$ $\int_a^b (g(x))^2\,dx = 3$

① $\int_a^b (f(x)+g(x))\,dx = 8+2 = 10$

② $\int_a^b c\,f(z)\,dz = c\int_a^b f(z)\,dz = 8c$

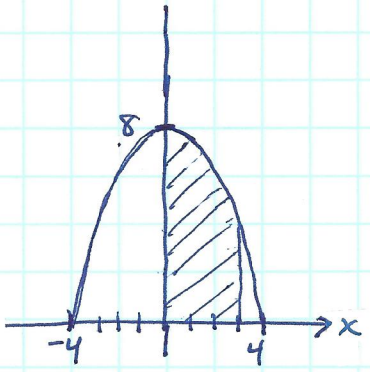
③ $\int_a^b (f(x))^2 - (g(x))^2\,dx = 12-3 = 9$

④ $\int_a^b f(x)^2\,dx - \left(\int_a^b f(x)\,dx\right)^2 = 12 - (8)^2 = 12-64 = -52$

⑤ $\int_a^b \left(c_1 g(x) + (c_2 f(x))^2\right) dx = c_1\int_a^b g(x)\,dx + (c_2)^2\left(\int_a^b f(x)\,dx\right)^2$

$= c_1(2) + (c_2)^2 12$

⑥ $\int_{a+5}^{b+5} f(x-5)\,dx = \int_a^b f(x)\,dx = 8$

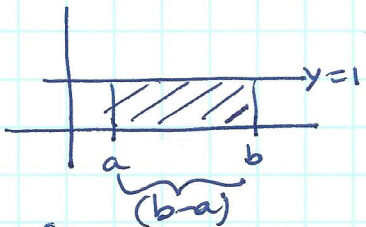


(22) $\int_0^3 f(x)\,dx = \frac{1}{2}\int_{-1}^{1} f(x)\,dx + \int_1^3 f(x)\,dx$

$= \int_0^1 f(x)\,dx + \int_1^3 f(x)\,dx$

(23) $\int_a^b 1\,dx$

$= (1)(b-a)$



b) i) $\int_2^5 (1)\,dx = 1(5-2) = 3$ ii) $\int_{-3}^{8} 1\,dx = 1(8+3) = 11$ iii) $\int_1^3 23\,dx$

$23\int_1^3 1\,dx$

$23(1)(3-1)$

$(23)(2) = 46$

(24) $\int_2^5 (2f(x)+3)\,dx = 17$

$2\int_2^5 f(x)\,dx + \int_2^5 3\,dx = 17$

$2\int_2^5 f(x)\,dx + (3)(5-2) = 17$

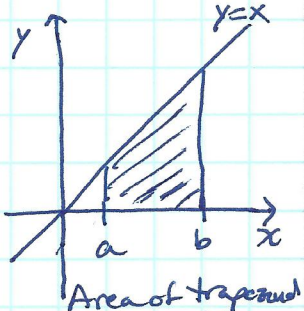
$2\int_2^5 f(x)\,dx = 8$

$\int_2^5 f(x)\,dx = 4$

(26) a) $\int_a^b x\,dx$

$= \frac{1}{2}(a+b)(b-a)$

$= \frac{1}{2}(b^2-a^2)$



Area of trapezoid $\frac{1}{2}(b_1+b_2)(h)$

b) i) $\int_2^5 x\,dx$

$= \frac{1}{2}(5^2-2^2) = \frac{1}{2}(21)$

ii) $\int_{-3}^{8} x\,dx$

$= \frac{1}{2}(8^2-(-3)^2) = \frac{1}{2}(64-9) = \frac{55}{2}$

iii) $\int_1^3 5x\,dx = 5\int_1^3 x\,dx$

$5\left(\frac{1}{2}(3^2-1^2)\right) = 5(4) = 20$

DAY 87 (D)

CH 5.4 PROPERTIES OF INTEGRALS & THEOREMS

p. 305-307 # 27, 28, 29, 31, 37

(27) If $f(x)$ is odd $\int_{-2}^{3} f(x)\,dx = 30$ find $\int_{2}^{3} f(x)\,dx = ?$

$\int_{-2}^{2} f(x)\,dx = 0 \quad \therefore \int_{2}^{3} f(x)\,dx = 30$

(28) If $f(x)$ is even $\int_{-2}^{2}(f(x)-3)\,dx = 8$ find $\int_{0}^{2} f(x)\,dx = ?$

$\int_{-2}^{2} f(x)\,dx + \int_{-2}^{2}(-3)\,dx = 8$

$\int_{-2}^{2} f(x)\,dx - 12 = 8$

$2\int_{0}^{2} f(x)\,dx = 20 \quad \therefore \int_{0}^{2} f(x)\,dx = 10$

odd function: $f(-x) = -f(x)$
even function: $f(-x) = f(x)$

(29) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \underbrace{x^3 \cos x^2\,dx}$ odd or even?

$f(-x) = (-x)^3 \cos(-x)^2$

$= -x^3 \cos x^2$

$= -f(x) \quad \therefore \quad x^3\cos x^2$ is <u>odd function</u> $\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 \cos x^2 = 0$

(31) $\int_{1}^{3} 3x^2\,dx = 26 \qquad \int_{1}^{3} 2x\,dx = 8$

$\int_{1}^{3} x^2\,dx = \frac{26}{3} \qquad \int_{1}^{3} x\,dx = 4$

$\therefore \int_{1}^{3}(x^2 - x)\,dx = \frac{26}{3} - 4 = \frac{14}{3}$

(?) (37) $2 \le \int_{0}^{2} \sqrt{1+x^3}\,dx \le 6$

$4 \le \int_{0}^{2} (1+x^3)\,dx \le 34$

$4 \le 2 + \int_{0}^{2} x^3\,dx \le 36$

$2 \le \int_{0}^{2} x^3\,dx \le 34$