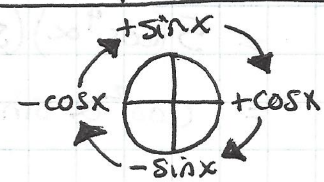
DAY 47

p. 153-154 #2, 3, 6, 7, 10, 11, 18, 19, 24, 36, 38, 42, 45, 54, 60

TRIG DERIVATIVES:

$\frac{d}{dx}(\sin x) = +\cos x$	$\frac{d}{dx}(\cos x) = -\sin x$ negative	$\frac{d}{dx}(\tan x) = +\sec^2 x$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$ negative	$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\cot x) = -\csc^2 x$ negative

NOTICE ALL $\frac{d}{dx}(c\ldots(x)) = -$ negative



CLOCKWISE derivatives.

② $r(\theta) = \sin\theta + \cos\theta$

$r'(\theta) = \cos\theta - \sin\theta$

③ $s(\theta) = (\cos\theta)(\sin\theta)$

$s'(\theta) = (-\sin\theta)(\sin\theta) + (\cos\theta)(\cos\theta)$

$s'(\theta) = -\sin^2\theta + \cos^2\theta$

$s'(\theta) = \cos^2\theta - \sin^2\theta$ Identity

$s'(\theta) = \cos(2\theta)$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

⑥ $y = 5\sin(3t)$

$\frac{dy}{dx} = 5 \cdot 3\cos(3t)$

$= 15\cos(3t)$

⑦ $P(t) = 4\cos(2t)$

$P'(t) = 4 \cdot 2(-\sin(2t))$

$P'(t) = -8\sin(2t)$

⑩ $g(\theta) = \sin^2(2\theta) - \pi\theta$

$g'(\theta) = 2\sin^1(2\theta) \cdot \cos(2\theta) \cdot 2 - \pi$

$= 4\sin(2\theta)\cos(2\theta) - \pi$ Identity

$= 2\sin(4\theta) - \pi$

$2\sin\theta\cos\theta = \sin 2\theta$

$\therefore 2\sin(2\theta)\cos(2\theta) = \sin(4\theta)$

⑪ $g(t) = (2 + \sin(\pi t))^3$

$g'(t) = 3(2 + \sin(\pi t))^2 \cdot (\pi\cos(\pi t))$

$g'(t) = 3\pi\cos(\pi t)[2 + \sin(\pi t)]^2$

⑱ $g(\theta) = \sin(\tan\theta)$

$g'(\theta) = \cos(\tan\theta) \cdot \sec^2\theta$

⑲ $w(x) = \tan(x^2)$

$w'(x) = \sec^2(x^2) \cdot 2x$

$w'(x) = 2x \cdot \sec^2(x^2)$

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24, 36, 38, 42, 45, 54, 60

(24) $k(x) = \sqrt{(\sin(2x))^3} = (\sin(2x))^{3/2}$

$k'(x) = \frac{3}{2}[\sin(2x)]^{\frac{1}{2}} \cdot \cos(2x) \cdot (2)$

$= 3 \cdot \sqrt{\sin(2x)} \cdot \cos(2x)$

(36) $k(\alpha) = \sin^5\alpha \cos^3\alpha = \boxed{(\sin\alpha)^5}\boxed{(\cos\alpha)^3}$

$k'(\alpha) = \boxed{5(\sin^4\alpha)(\cos\alpha)}\boxed{(\cos^3\alpha)} + \boxed{3(\cos^2\alpha)(-\sin\alpha)}\boxed{\sin^5\alpha}$

$= 5(\cos^4\alpha)(\sin^4\alpha) - 3(\cos^2\alpha)(\sin^6\alpha)$

$= \cos^2\alpha \sin^4\alpha\,[5\cos^2\alpha - 3\sin^2\alpha]$ GCF factored.

(38) $y = \cos^2 w + \cos(w^2)$

$\frac{dy}{dx} = 2\cos^1(w) + -\sin(w^2)(2w)$

$= 2(\cos(w) - w\sin(w^2))$

(42) $t(\theta) = \frac{\cos\theta}{\sin\theta} = \cot\theta$ $\quad d(\cot\theta) = -\csc^2\theta$

$t'(\theta) = \frac{\sin\theta(-\sin\theta) - \cos\theta(\cos\theta)}{\sin^2\theta} = \frac{-(\sin^2\theta + \cos^2\theta)}{\sin^2\theta}$

$= \frac{-1}{\sin^2\theta} = -\csc^2\theta$

(45) $G(x) = \frac{\sin^2 x + 1}{\cos^2 x + 1}$ $\quad G'(x) = \frac{(\cos^2 x + 1)(2\sin x\cos x) - (\sin^2 x + 1)(-2\cos x\sin x)}{[\cos^2 x + 1]^2}$

$= \frac{(2\sin x\cos x)[\cos^2 x + 1 + \sin^2 x + 1]}{[\cos^2 x + 1]^2}$

Identity

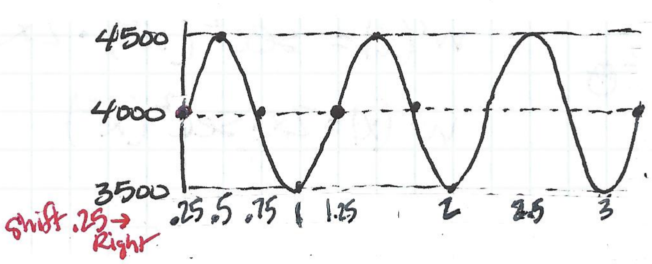
$= \frac{(2\sin x\cos x)(3)}{[\cos^2 x + 1]^2} = \frac{3\sin(2x)}{[\cos^2 x + 1]^2}$

(54) $f(x) = \sin^2 x + \cos^2 x = 1 \rightarrow \frac{dy}{dx} = 0$

$f'(x) = 2\sin x\cos x + 2\cos x(-\sin x)$

$f'(x) = 0$

(60) $P(t) = 4000 + 500\sin\left(2\pi t - \frac{\pi}{2}\right)$



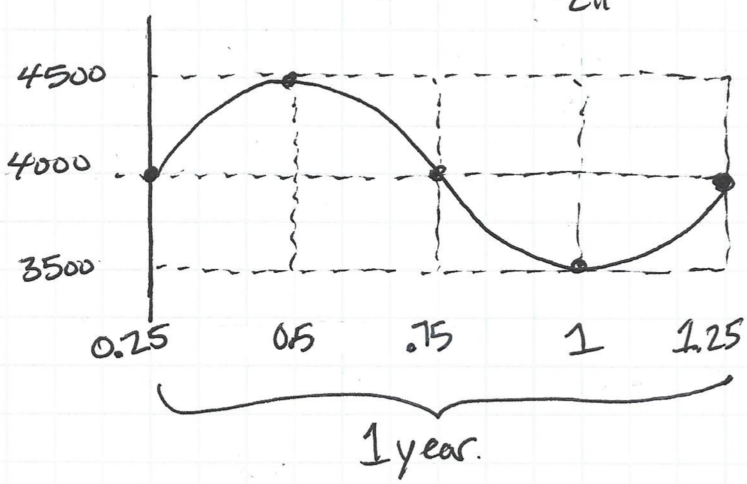
see next page #60

$\boxed{\text{DAY 47}}$ continued

(60) $P(t) = 4000 + 500\sin\left(2\pi t - \frac{\pi}{2}\right)$

$P(t) = 4000 + 500\sin\left(2\pi\left(t - \frac{1}{4}\right)\right)$

a) sinusoidally. period $= \frac{2\pi}{2\pi} = 1$ $\therefore$ 1 year = 1 cycle.



b) $P'(t) = 500\cos\left(2\pi\left(t - \frac{1}{4}\right)\right) \cdot 2\pi$

$P'(t) = 1000\pi\cos\left(2\pi\left(t - \frac{1}{4}\right)\right)$

$P(t)$ has a MAX or MIN when $P'(t) = 0$ ($\therefore$ $P(t)$ has a horizontal tangent)

$P'(t) = 0 = 1000\pi\cos\left(2\pi\left(t - \frac{1}{4}\right)\right)$

$\cos\left(2\pi\left(t - \frac{1}{4}\right)\right) = 0$

$2\pi\left(t - \frac{1}{4}\right) \in \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\} + 2\pi k$

$\left(t - \frac{1}{4}\right) \in \left\{\frac{1}{4}, \frac{3}{4}\right\} + 1k$

$t \in \left\{\frac{1}{2}, 1\right\} + 1k.$

$P\left(\frac{1}{2}\right) = \text{MAX} = 4500$

$P(1) = \text{MIN} = 3500$

2nd deriv.

c) $P''(t) = -2000\pi^2\sin\left(2\pi\left(t - \frac{1}{4}\right)\right) = 0$

$\sin\left(2\pi\left(t - \frac{1}{4}\right)\right) = 0$

$2\pi\left(t - \frac{1}{4}\right) \in \{0, \pi\} + 2\pi k$

$\left(t - \frac{1}{4}\right) \in \left\{0, \frac{1}{2}\right\} + 1k$

$t \in \left\{\frac{1}{4}, \frac{3}{4}\right\} + 1k$

$P\left(\frac{1}{4}\right) \Rightarrow$ population is increasing the fastest Point of Inflection $\left(\frac{1}{4}, 4000\right)$

$P\left(\frac{3}{4}\right) \Rightarrow$ population is decreasing the fastest " " " $\left(\frac{3}{4}, 4000\right)$