$\boxed{\text{DAY 41}}$ § 3.2 The Exponential Function

$$\boxed{\begin{aligned} y &= a\cdot b^x \\ \frac{dy}{dx} &= a\cdot \ln(b)\cdot b^x \\ \frac{dy}{dx} &= \ln(b^a)\cdot b^x \end{aligned}}$$

NOTE: $\boxed{\ln(e)=1 \quad \ln(0) \text{ dne} \quad \ln(1)=0}$

p. 135-136
\# 3-24 (x3) 38, 39, 42, 43, 45

(3) $f(x) = a^{5x} = (a^5)^x$

$f'(x) = \ln(a^5)\cdot a^{5x}$

(6) $f(x) = 2^x + 2\cdot 3^x$

$f'(x) = \ln(2)\cdot 2^x + 2\cdot \ln(3)\cdot 3^x$

$f'(x) = \ln(2)\cdot 2^x + \ln(9)\cdot 3^x$

(9) $y = \dfrac{3^x}{3} + \dfrac{33}{\sqrt{x}}$

$y = \frac{1}{3}\cdot 3^x + 33x^{-1/2}$

$\dfrac{dy}{dx} = \frac{1}{3}\ln(3)\cdot 3^x - \frac{33}{2}x^{-3/2}$

$\dfrac{dy}{dx} = \ln(\sqrt[3]{3})\cdot 3^x - \dfrac{33}{2x^{3/2}}$

(12) $f(x) = (\ln 3)^x$

$f'(x) = \ln(\ln 3)\cdot(\ln 3)^x$

(15) $f(x) = e^2 + x^e$

$f'(x) = 0 + ex^{e-1} = ex^{e-1}$

(18) $f(x) = \pi^x + x^\pi$

$f'(x) = (\ln\pi)\pi^x + \pi x^{\pi-1}$

(21) $f(x) = e^{x+2} = e^2\cdot e^x$

$f'(x) = e^2(\ln e)e^x$

$f'(x) = e^2\cdot e^x = e^{x+2}$

(24) $f(x) = x^{\pi^2} + (\pi^2)^x$

$f'(x) = \pi^2\left(x^{\pi^2-1}\right) + \ln(\pi^2)(\pi^2)^x$

$\boxed{\begin{aligned} f(x) &= e^x \\ f'(x) &= e^x \end{aligned}}$ $\boxed{\begin{aligned} f(x) &= e^{x\pm c} \\ f'(x) &= e^{x\pm c} \end{aligned}}$

(38) $P(t) = 300(1.044)^t$ $P(t)$ = Animal population, t = # years since study began.

$P'(t) = 300(\ln 1.044)(1.044)^t$

$P'(5) = 300(\ln 1.044)\cdot(1.044)^5 = 16.0211 \ \frac{\text{animals}}{\text{year}}$

$\underbrace{\text{In the 5th year after the study began}}_{\text{WHEN}}$, $\underbrace{\text{the animal population}}_{\text{WHAT}}$ $\underbrace{\text{is increasing}}_{\text{BEHAVIOR}}$ at a $\underbrace{\text{rate of 16}}_{\text{rate value}}$ $\underbrace{\text{animals/year}}_{\text{units}}$.

$\boxed{\text{DAY 41}}$ continued.

39, 42, 43, 45

(39) $P(t) = P_0(1.05)^t$ $P(t)$ = price after inflation.
P_0 = price in \$ t = time in years.

$P'(t) = P_0\,(\ln(1.05))\,(1.05)^t$

$P'(10) = P_0\,(\ln 1.05)\,(1.05)^{10}$ after 10 years

$P'(10) = (P_0)(0.07947)$

$P'(10) \approx (0.0795)(P_0)$

When/What/Behavior/Rate Value/Units

$\boxed{\text{When}}$: After 10 years $\boxed{\text{What}}$: prices $\boxed{\text{behavior}}$: are increasing

$\boxed{\text{RATE:}}$ at a rate of \$0.0795/year OR $7.95\ \dfrac{\text{cents}}{\text{year}}$

$\boxed{\text{UNITS}}$ $\boxed{\text{UNITS.}}$

(42) $V(t) = 75(1.35)^t$ Price \$ of Rocking chair since 2000.

$V'(t) = 75\,(\ln(1.35))(1.35)^t$ $V'(t): \dfrac{\$}{\text{year}}$

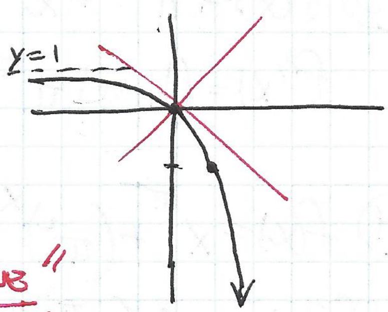
(43) $f(x) = 1 - e^x$ $(0,0)$ $1 - e^x = 0$ $f(x)$ crosses x-axis at $x=0$

$1 = e^x$

$\ln(1) = x$

$x = 0$

$f'(x) = -e^x$



a) $f'(0) = -e^0 = -1$

b) $y = -1(x-0)+0$ or $\boxed{y = -1x}$

c) Line $\perp$ to $\boxed{\text{tangent line}}$ = "NORMAL LINE"

$\boxed{y = 1x}$

(45) $g(x) = ax^2 + bx + c$ the best fit function to $f(x) = e^x$ @ $x=0$

$g(0) = f(0)$ $g'(0) = f'(0)$ $g''(0) = f''(0)$

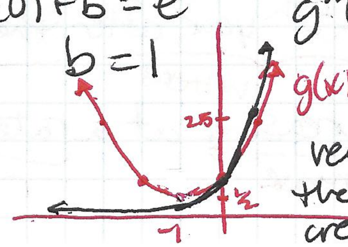
$a(0)^2 + b(0) + c = e^0$ $g'(x) = 2ax + b = e^x$ $g''(x) = 2a = e^x$

$\therefore c = 1$ $g'(0) = 2a(0) + b = e^0$ $g''(0) = 2a = 1$

$b = 1$ $a = \frac{1}{2}$

$\therefore g(x) = \frac{1}{2}x^2 + 1x + 1$

$\approx f(x) = e^x$ near $x=0$



very close to x=0 the two graphs are about the same.