$\boxed{\text{DAY 40}}$ § 3.1 Power Rule

$$\boxed{y = ax^n \quad \frac{dy}{dx} = a \cdot n \cdot x^{n-1}}$$

P. 129–130
38–48 even, 56, 58, 62, 68, 83–90

(38) $y = \frac{x^2+1}{x} = x + x^{-1}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2} = \frac{x^2-1}{x}$

(40) $g(x) = \frac{x^2+\sqrt{x}+1}{x^{3/2}} = x^{1/2} + x^{-1} + x^{-3/2}$

$g'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2} - \frac{3}{2x^{5/2}}$

$g'(x) = \frac{x^2 - 2\sqrt{x} - 3}{2x^{5/2}}$

(42) $g(x) = \frac{\sqrt{x}\,(1+x)}{x^2} = \frac{x^{1/2}+x^{3/2}}{x^2} = x^{-3/2} + x^{-1/2}$

$g'(x) = -\frac{3}{2}x^{-5/2} - \frac{1}{2}x^{-3/2} = \frac{-3}{2x^{5/2}} - \frac{1}{2x^{3/2}} = \frac{-3-x}{2x^{5/2}}$

(44) $f(x) = \frac{ax+b}{x} = a + bx^{-1}$

$f'(x) = \frac{-b}{x^2}$

(46) $V(r) = \left(\frac{4\pi b}{3}\right) r^2$

$V'(r) = \left(\frac{4\pi}{3}b\right) 2r^1$

$V'(r) = \frac{8\pi b}{3} \cdot r^1$

(48) $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

(56) $f(x) = x^3 - 9x^2 - 16x + 1$
has slope of $f'(x) = 5$
at two points. . . .

$f'(x) = 3x^2 - 18x - 16 = 5$

$3x^2 - 18x - 21 = 0$

$3(x^2 - 6x - 7) = 0$

$3(x-7)(x+1) = 0$

$x = 7 \quad x = -1$

@ TWO POINTS: $\underline{(7, -209)}$ $\underline{(-1, 7)}$

(58) $f(x) = x^3$ tangent line @ $x = 2$

$f(2) = 8 \quad \therefore \quad (2, 8)$

$f'(x) = 3x^2\Big|_{x=2} = 12$

TANGENT LINE:

$\underline{y = 12(x-2) + 8}$

(88) The derivative of a polynomial is always a polynomial
TRUE.

(89) $\frac{d}{dx}\left(\frac{\pi}{x^2}\right) = \frac{-\pi}{x}$

FALSE it equals $\left(\frac{-2\pi}{x^3}\right)$

(90) If $f'(2) = 3.1$ $g'(2) = 7.3$
then $\frac{d}{dx}\left(f(x) + g(x)\right)\Big|_{x=2} = 10.4$

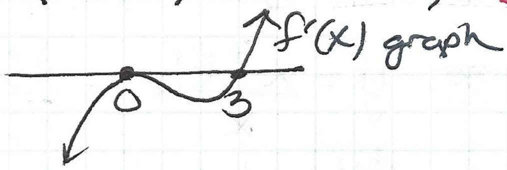
TRUE.

DAY 40 continued

★ (62) $f(x) = x^4 - 4x^3$ is both decreasing & ccup on what intervals.

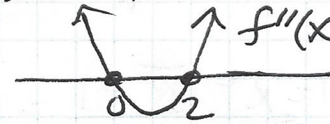
$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) = 0 \quad x = 0, 3$

$f'(x) < 0 \Rightarrow f$ is decreasing on $(-\infty, 0)(0, 3)$



$f''(x) = 12x^2 - 24x = 12x(x-2) = 0 \quad x = 0, 2$

$f''(x) > 0 \Rightarrow f$ is concave up. on $(-\infty, 0)(2, \infty)$



Oh... now I need to apply poly inequality ideas.

$\therefore$ $\boxed{f(x) \text{ is decreasing and concave up on } x \in (-\infty, 0)\ (2, 3) \text{ when } f'(x) < 0\ \&\ f''(x) > 0}$

(68) height of sand dune (in cm) $f(t) = 700 - 3t^2$ where t measured in years since 2005.

$f(5) = 700 - 3(5)^2 = 625$ cm.

★ In 2010 the height of the sanddune is 625 cm.

$f'(5)$: $f'(t) = -6t\big|_{t=5} = -30 \dfrac{\text{cm}}{\text{year}} = f'(5)$

★ In the year 2010 the height of the sand dune is decreasing at a rate of 30 cm/year.

NEED TO SEE: ★ when / what / behavior / rate value / units !

(83) The only fn that has derivatie $2x$ is x^2 FALSE counter-examples: $\boxed{f(x) = x^2 + 3 \quad f(x) = x^2 - 7}$

(84) $f(x) = \frac{1}{x^2} \to f'(x) = \frac{1}{2x}$ FALSE $\boxed{f(x) = x^{-2} \quad f'(x) = -2x^{-3} = \frac{-2}{x^3}}$

(85) Two functions s.t. $\frac{d}{dx}\left(f(x) + g(x)\right) = 2x + 3$

(A) $f(x) + g(x) = x^2 + 3x$ (B) $f(x) + g(x) = (x^2 + 7) + (3x - 1)$

(86) A function whose derivative is $g'(x) = 2x$ & $g(x)$ has no zeros

$g(x) = x^2 + 3$ or $g(x) = x^2 + 5$ or $g(x) = x^2 + c$, where $c > 0$

(87) $f''(x) = 6 \Rightarrow f'(x) = 6x + c \Rightarrow f(x) = 3x^2 + cx + d$
$c \in \mathbb{R}$ $\quad c \in \mathbb{R}\ d \in \mathbb{R}$

Go back to front for 88–90 →