HW (ANSWERS ONLY HERE) ... SHOW WORK TO FIND ANSWERS!!

AB REVIEW #1 FRQ

a) $y = \frac{-1}{2}(x-2)+3$

b) Area $= 16$

AB REVIEW #2 FRQ

a) ht $= 12$ cm

b) $\frac{dh}{dt} = 5$cm

AB REVIEW #3 FRQ

a) domain $x \neq \pm 2$

b) VA $x = \pm 2$

HA $y = 0$

c) $f'(x) = \frac{-2x^2+10x-8}{(x^2-4)^2} = \frac{-2(x-4)(x-1)}{(x^2-4)^2}$

d) $y = -\frac{1}{2}(x-0) + \frac{5}{4}$

AB REVIEW #4 FRQ

a) $A = \int_0^{\ln 4} (e^x - e^{-x})\,dx = \ldots = \frac{9}{4}$

b) $V = \pi \int_0^{\ln(4)} (e^{2x} - e^{-2x})\,dx$

c) $V = \int_{1/4}^{4} (\ln 4)^2 - (\ln y)^2\,dy$

AB REVIEW #5 FRQ

a) $x \in \{0, \frac{\pi}{2}\} + \pi k$

b) NONE b/c $f'(x) > 0$ for all x

c) $x \in \{\frac{\pi}{4}, \frac{3\pi}{4}\} + \pi k$

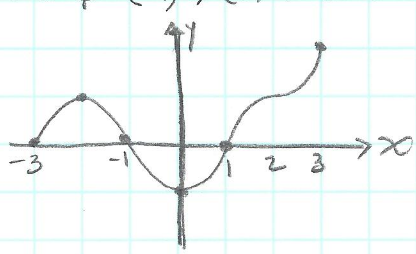
AB REVIEW #6 FRQ

a) Rel Max @ $x = -2$ b/c f' changes signs (+) to (−).

Rel Min @ $x = 0$ b/c f' changes signs (−) to (+)

b) f ccup on $(-1,1)$ $(2,3)$ b/c f' is increasing

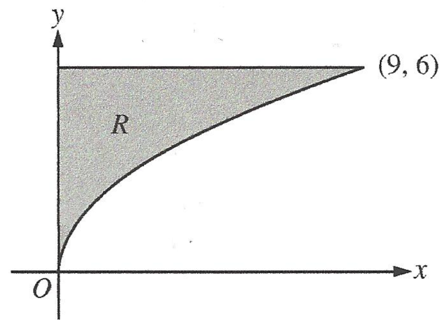
c)



A AREAS & VOLUMES

AP® CALCULUS AB
[redacted] SCORING GUIDELINES

Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) Area $= \int_0^9 \left(6 - 2\sqrt{x}\right)\,dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big\vert_{x=0}^{x=9} = 18$	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi\int_0^9 \left(\left(7 - 2\sqrt{x}\right)^2 - (7-6)^2\right)dx$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$
(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$. Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$. Volume $= \int_0^6 \frac{3}{16}y^4\,dy$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

B AVERAGE & INSTANTANEOUS RATES OF CHANGE

AP® CALCULUS AB
[redacted] SCORING GUIDELINES (Form B)

Question 2

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

1 : shows that $R(6) > 0$

(b) $R'(6) = -1.913$

Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

$2 : \begin{cases} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{cases}$

(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$

To the nearest whole number, there are 964 mosquitoes.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$

$R(t) > 0$ on $0 < t < 2.5\pi$

$R(t) < 0$ on $2.5\pi < t < 7.5\pi$

$R(t) > 0$ on $7.5\pi < t < 31$

The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

$1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357$,

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

4 :
- 2 : absolute maximum value
  - 1 : integral
  - 1 : answer
- 2 : analysis
  - 1 : computes interior critical points
  - 1 : completes analysis

B | AVERAGE & INSTANTANEOUS RATES OF CHANGE

AP® CALCULUS AB

[redacted] SCORING GUIDELINES (Form B)

Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

(a) $a(5) \approx \dfrac{v(10) - v(0)}{10 - 0} = \dfrac{0.3}{10} = 0.03 \text{ meters/sec}^2$

1 : answer

(b) $\int_0^{60} |v(t)|\, dt$ is the total distance, in meters, that Ben rides over the 60-second interval $t = 0$ to $t = 60$.

$\int_0^{60} |v(t)|\, dt \approx 2.0 \cdot 10 + 2.3(40 - 10) + 2.5(60 - 40) = 139$ meters

$2 : \begin{cases} 1 : \text{meaning of integral} \\ 1 : \text{approximation} \end{cases}$

(c) Because $\dfrac{B(60) - B(40)}{60 - 40} = \dfrac{49 - 9}{20} = 2$, the Mean Value Theorem implies there is a time t, $40 < t < 60$, such that $v(t) = 2$.

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{conclusion with justification} \end{cases}$

(d) $2L(t)L'(t) = 2B(t)B'(t)$

$L'(40) = \dfrac{B(40)v(40)}{L(40)} = \dfrac{9 \cdot 2.5}{\sqrt{144 + 81}} = \dfrac{3}{2}$ meters/sec

$3 : \begin{cases} 1 : \text{derivatives} \\ 1 : \text{uses } B'(t) = v(t) \\ 1 : \text{answer} \end{cases}$

1 : units in (a) or (b)

C BIG THEOREMS

AP® CALCULUS AB
[redacted] SCORING GUIDELINES (Form B)

Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\,dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\,dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_0^{30} a(t)\,dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t)\,dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

(a) $\int_{30}^{60} |v(t)|\,dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)|\,dt$:

$$A = \frac{1}{2}(14+10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

$2: \begin{cases} 1: \text{explanation} \\ 1: \text{value} \end{cases}$

(b) $\int_0^{30} a(t)\,dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\int_0^{30} a(t)\,dt = \int_0^{30} v'(t)\,dt = v(30) - v(0)$$
$$= -14 - (-20) = 6 \text{ ft/sec}$$

$2: \begin{cases} 1: \text{explanation} \\ 1: \text{value} \end{cases}$

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

$2: \begin{cases} 1: v(35) < -5 < v(50) \\ 1: \text{Yes; refers to IVT or hypotheses} \end{cases}$

(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

$2: \begin{cases} 1: v(0) = v(25) \\ 1: \text{Yes; refers to MVT or hypotheses} \end{cases}$

Units of ft in (a) and ft/sec in (b)

1 : units in (a) and (b)

D DIFFERENTIAL EQUATIONS

AP® CALCULUS AB
[redacted] SCORING GUIDELINES

Question 6

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f'(1) = \left.\frac{dy}{dx}\right|_{(1,\,2)} = 8$

An equation of the tangent line is $y = 2 + 8(x - 1)$.

$2: \begin{cases} 1 : f'(1) \\ 1 : \text{answer} \end{cases}$

(b) $f(1.1) \approx 2.8$

Since $y = f(x) > 0$ on the interval $1 \le x < 1.1$,

$\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right) > 0$ on this interval.

Therefore on the interval $1 < x < 1.1$, the line tangent to the graph of $y = f(x)$ at $x = 1$ lies below the curve and the approximation 2.8 is less than $f(1.1)$.

$2: \begin{cases} 1 : \text{approximation} \\ 1 : \text{conclusion with explanation} \end{cases}$

(c) $\frac{dy}{dx} = xy^3$

$\int \frac{1}{y^3}\,dy = \int x\,dx$

$-\frac{1}{2y^2} = \frac{x^2}{2} + C$

$-\frac{1}{2 \cdot 2^2} = \frac{1^2}{2} + C \Rightarrow C = -\frac{5}{8}$

$y^2 = \frac{1}{\frac{5}{4} - x^2}$

$f(x) = \frac{2}{\sqrt{5 - 4x^2}}, \quad \frac{-\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$

$5: \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

E INTEGRATION TECHNIQUES

AP® CALCULUS AB
[redacted] SCORING GUIDELINES (Form B)

Question 6

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

(a) Find $f''(3)$.

(b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

(a) $f''(x) = \sqrt{f(x)} + x \cdot \dfrac{f'(x)}{2\sqrt{f(x)}} = \sqrt{f(x)} + \dfrac{x^2}{2}$

$f''(3) = \sqrt{25} + \dfrac{9}{2} = \dfrac{19}{2}$

3 : $\begin{cases} 2 : f''(x) \\ \quad < -2 > \text{ product or chain rule error} \\ 1 : \text{value at } x = 3 \end{cases}$

(b) $\dfrac{1}{\sqrt{y}}\,dy = x\,dx$

$2\sqrt{y} = \dfrac{1}{2}x^2 + C$

$2\sqrt{25} = \dfrac{1}{2}(3)^2 + C$; $C = \dfrac{11}{2}$

$\sqrt{y} = \dfrac{1}{4}x^2 + \dfrac{11}{4}$

$y = \left(\dfrac{1}{4}x^2 + \dfrac{11}{4}\right)^2 = \dfrac{1}{16}\left(x^2 + 11\right)^2$

6 : $\begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } f(3) = 25 \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

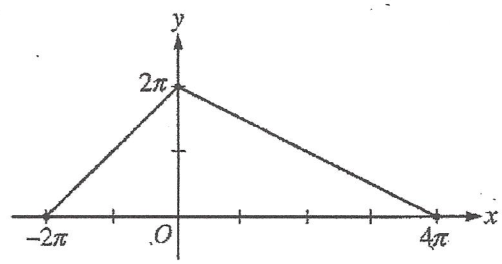
Note: 0/6 if no separation of variables

F INTERPRETING GRAPHS

AP® CALCULUS AB
[redacted] SCORING GUIDELINES (Form B)

Question 6

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.



Graph of g

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

(a) $\int_{-2\pi}^{4\pi} f(x)\,dx = \int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\frac{x}{2}\right)\right) dx$

$$= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right)\right]_{x=-2\pi}^{x=4\pi}$$

$$= 6\pi^2$$

$2: \begin{cases} 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

(b) $f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$

$f'(x)$ does not exist at $x = 0$.
For $-2\pi < x < 0$, $f'(x) \neq 0$.
For $0 < x < 4\pi$, $f'(x) = 0$ when $x = \pi$.

f has critical points at $x = 0$ and $x = \pi$.

$4: \begin{cases} 1: \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right) \\ 1: g'(x) \\ 1: x = 0 \\ 1: x = \pi \end{cases}$

(c) $h'(x) = g(3x) \cdot 3$

$h'\left(-\frac{\pi}{3}\right) = 3g(-\pi) = 3\pi$

$3: \begin{cases} 2: h'(x) \\ 1: \text{answer} \end{cases}$

F INTERPRETING GRAPHS

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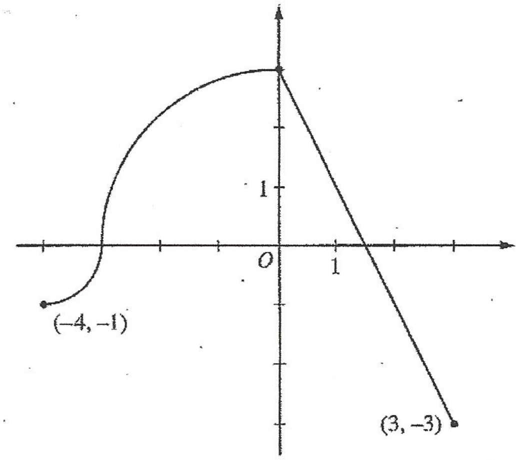
[redacted] SCORING GUIDELINES

Question 4

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.



Graph of f

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3) = 2$

$3: \begin{cases} 1: g(-3) \\ 1: g'(x) \\ 1: g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.

$g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.

Therefore g has an absolute maximum at $x = \frac{5}{2}$.

$3: \begin{cases} 1: \text{considers } g'(x) = 0 \\ 1: \text{identifies interior candidate} \\ 1: \text{answer with justification} \end{cases}$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \le x \le 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

$2: \begin{cases} 1: \text{average rate of change} \\ 1: \text{explanation} \end{cases}$

G PARTICLE MOTION

AP® CALCULUS AB
[redacted] SCORING GUIDELINES

Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337,\ a(5.5) = -1.35851$ The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.	2 : conclusion with reason
(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$	2 : { 1 : integral 1 : answer
(c) Distance $= \int_0^6 \lvert v(t)\rvert\,dt = 12.573$	2 : { 1 : integral 1 : answer
(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$. $v(t)$ changes sign from positive to negative at time $t = b$. $x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135	3 : { 1 : considers $v(t) = 0$ 1 : integral 1 : answer

H RATES & ACCUMULATION

AP® CALCULUS AB
[redacted] SCORING GUIDELINES (Form B)

Question 2

A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

(a) $\lim\limits_{t \to 5^-} r(t) = \lim\limits_{t \to 5^-} \left(\dfrac{600t}{t+3}\right) = 375 = r(5)$

$\lim\limits_{t \to 5^+} r(t) = \lim\limits_{t \to 5^+} \left(1000e^{-0.2t}\right) = 367.879$

Because the left-hand and right-hand limits are not equal, r is not continuous at $t = 5$.

2 : conclusion with analysis

(b) $\dfrac{1}{8}\displaystyle\int_0^8 r(t)\,dt = \dfrac{1}{8}\left(\int_0^5 \dfrac{600t}{t+3}\,dt + \int_5^8 1000e^{-0.2t}\,dt\right)$

$= 258.052$ or 258.053

3 : { 1 : integrand; 1 : limits and constant; 1 : answer }

(c) $r'(3) = 50$

The rate at which water is draining out of the tank at time $t = 3$ hours is increasing at 50 liters/hour^2.

2 : { 1 : $r'(3)$; 1 : meaning of $r'(3)$ }

(d) $12{,}000 - \displaystyle\int_0^A r(t)\,dt = 9000$

2 : { 1 : integral; 1 : equation }

I RELATED RATES

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000 \text{ cm}^3/\text{min}$ and $\frac{dr}{dt} = 2.5 \text{ cm/min}$.

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.038 \text{ or } 0.039 \text{ cm/min}$$

4 : { 1 : $\frac{dV}{dt} = 2000$ and $\frac{dr}{dt} = 2.5$; 2 : expression for $\frac{dV}{dt}$; 1 : answer }

(b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $R(t) = 2000$.
This occurs when $t = 25$ minutes.
Since $\frac{dV}{dt} > 0$ for $0 < t < 25$ and $\frac{dV}{dt} < 0$ for $t > 25$, the oil slick reaches its maximum volume 25 minutes after the device begins working.

3 : { 1 : $R(t) = 2000$; 1 : answer; 1 : justification }

(c) The volume of oil, in cm^3, in the slick at time $t = 25$ minutes is given by $60{,}000 + \int_0^{25} (2000 - R(t))\, dt$.

2 : { 1 : limits and initial condition; 1 : integrand }

J TABLES

AP® CALCULUS AB

[redacted] SCORING GUIDELINES

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a) $$H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2} = \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

1 : answer

(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(2\cdot\frac{66+60}{2} + 3\cdot\frac{60+52}{2} + 4\cdot\frac{52+44}{2} + 1\cdot\frac{44+43}{2}\right) = 52.95$$

3 : { 1 : meaning of expression; 1 : trapezoidal sum; 1 : estimate }

(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : { 1 : value of integral; 1 : meaning of expression }

(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275;\quad H(10) - B(10) = 8.817$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : { 1 : integrand; 1 : uses $B(0) = 100$; 1 : answer }

J TABLES

Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

(a) $\int_0^{1.5} (3f'(x) + 4)\,dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$

$= 3f(x) + 4x\Big|_0^{1.5} = 3(-1 - (-7)) + 4(1.5) = 24$

2 { 1 : antiderivative; 1 : answer }

(b) $y = 5(x - 1) - 4$

$f(1.2) \approx 5(0.2) - 4 = -3$

The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.

3 { 1 : tangent line; 1 : computes y on tangent line at $x = 1.2$; 1 : answer with reason }

(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that

$f''(c) = \dfrac{f'(0.5) - f'(0)}{0.5 - 0} = \dfrac{3 - 0}{0.5} = 6 = r$

2 { 1 : reference to MVT for f' (or differentiability of f'); 1 : value of r for interval $0 \le x \le 0.5$ }

(d) $\lim_{x \to 0^-} g'(x) = \lim_{x \to 0^-} (4x - 1) = -1$

$\lim_{x \to 0^+} g'(x) = \lim_{x \to 0^+} (4x + 1) = +1$

Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$.

OR

$g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.

2 { 1 : answers "no" with reference to g' or g''; 1 : correct reason }