

2008 AB Calculus #2:
CALCULATOR ACTIVE

Concert Tickets

Concert ticket went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function $L(t)$ for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?

(a) $L'(5.5) \approx \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = 8$ people per hour

2 : $\left\{ \begin{array}{l} 1 : \text{estimate} \\ 1 : \text{units} \end{array} \right.$

(b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4} \left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3) \right) = 155.25 \text{ people}$$

2 : $\left\{ \begin{array}{l} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{array} \right.$

(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

3 : $\left\{ \begin{array}{l} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{array} \right.$

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

OR

3 : $\left\{ \begin{array}{l} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{array} \right.$

[Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t .]

(d) $\int_0^3 r(t) dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

2 : $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{array} \right.$