§3.1 - §3.5 Applying the Derivative Rules using Tables

The purpose of this worksheet is to abstract the concept of the derivative rules by causing you to apply them to functions that you do not know. Two functions, $f(x)$ and $g(x)$, have the values and first derivatives shown in the table. Use this information to find the following.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-4	2	-2	-1	1
-3	1	-1	-2	2
-2	-2	1	0	3
-1	-1	4	2	1
0	0	5	1	0
1	2	3	0	-2
2	3	2	1	-1
3	3	1	-1	-3
4	1	-1	-2	-4

1. $h(x) = f(x) - g(x)$
   Find $h'(2)$

2. $h(x) = f(x) + 3g(x)$
   Find $h'(0)$

3. $h(x) = 2f(x) - 4g(x)$
   Find $h'(-3)$

4. $h(x) = 2f(x) - 1$
   Find $h'(3)$

5. $h(x) = 3g(x) - x^2$
   Find $h'(1)$

6. $h(x) = x\ f(x)$
   Find $h'(-1)$

7. $h(x) = [f(x)]^2$
   Find $h'(-3)$

8. $h(x) = f(x)g(x)$
   Find $h'(2)$

9. $h(x) = x^2 f(x)g(x)$
   Find $h'(-1)$

10. $h(x) = f(x)/g(x)$
    Find $h'(-2)$

11. $h(x) = f(3x)$
    Find $h'(-1)$

12. $h(x) = g\left(x^2\right)$
    Find $h'(-2)$

13. $h(x) = f\left(x^3 - x\right)$
    Find $h'(1)$

14. $h(x) = f(g(x))$
    Find $h'(4)$

15. $h(x) = g(f(x))$
    Find $h'(-3)$

16. $h(x) = [f(x)]^3\ g(-2x)$
    Find $h'(2)$

17. $h(x) = x^2/f(x)$
    Find $h'(-1)$

18. $h(x) = f(\ln\ x)/g(2x+1)$
    Find $h'(1)$

	ON YOUR PAPER	*ON CALCULATOR*

1 Given the function $f(x)$, **show the calculation** necessary to find the:

a) average rate of change, average velocity,
or slope of the secant on the interval $x \in [4.3, 5.6]$ ....>

b) instantaneous rate of change, instantaneous velocity,
or slope of the tangent line at $x = 4.95$ ....>

2 The height of a projectile propelled from a platform 120 feet in the air with an initial velocity of 96 ft/sec is given by the function $h(t) = -\frac{1}{2}a_0 t^2 + v_0 t + h_0$. Note: Earth's gravitational constant is 32 ft/sec^2.

Write the equation for $h(t)$ =_______________________________ and **show the calculation** necessary to find the:

a) average rate of change, average velocity, or slope of the secant on each of the intervals

Interval	$t \in [0,1]$	$t \in [1,2]$	$t \in [2,3]$	$t \in [3.012, 5.789]$	$t \in [4.218, 6.357]$
Algebraic Expression in terms of h(t)					

Since function is defined, write the expression for the slope of secant and evaluate the expression on the calculator. Record 3-decimal accuracy.

Evaluation (3-decimal accuracy)					

b) instantaneous rate of change, instantaneous velocity,
or slope of the tangent line at $t = 3.724$ seconds..........>

c) Examine the values for the first three intervals what do they tell you about the behavior of the function. You should be able to conclude two specific ideas.

3 Given the table of values

$x_{(sec)}$	0	1	2	3	4	5	6	7
$f(x)_{(meters)}$	120	200	248	264	248	200	120	8

show the calculation necessary to find the:

ON PAPER: must pull values from the table and use in calculation

a) average rate of change, average velocity,
or slope of the secant on the interval $x \in [4, 6]$ ....>

b) instantaneous rate of change, instantaneous velocity,
or slope of the tangent line at $x = 5$ ....>

Using appropriate **MATHEMATICAL NOTATION** to write what is required to justify Continuity & Differentiability.

4 Definition of Continuity in 3 parts.

a)

b)

c)

5 Definition of Differentibility.

§3.7 Implicit Differentiation -- A Brief Introduction -- Student Notes

Find these derivatives of these functions:

$y=\tan(x)$ $\quad\quad$ $y=\sin(x)$ $\quad\quad$ $y=e^x$

$\frac{d}{dx}(\tan(x))=$ $\quad\quad$ $\frac{d}{dx}(\sin(x))=$ $\quad\quad$ $\frac{d}{dx}(e^x)=$

Write the inverses of these functions:

$y=\tan(x)$ $\quad\quad$ $y=\sin(x)$ $\quad\quad$ $y=e^x$

How would we find the derivatives of these inverse functions?

$\frac{d}{dx}(\_\_\_\_\_\_\_\_\_\_)$ $\quad\quad$ $\frac{d}{dx}(\_\_\_\_\_\_\_\_\_\_)$ $\quad\quad$ $\frac{d}{dx}(\_\_\_\_\_\_\_\_\_\_)$

Let's look at a brief introduction to Implicit Differentiation so that we can find the derivatives of these three inverse functions.

Up to now, we have worked explicitly, solving an equation for one variable *y* in terms of another variable *x*. For example, if you were asked to find $\frac{dy}{dx}$ for $2x^2+y^2=4$, you would solve for *y* and get $y=\pm\sqrt{4-2x^2}$ and then take the derivative. This derivative requires the use of the chain rule.

Sometimes it is inconvenient, difficult or impossible to solve for *y*. In this case, we use implicit differentiation. It is imperative to note that anytime you see a *y*-variable you must think of *y* as a function of *x* just as in the notation: $y=f(x)$. Since I do not know the explicit form of *f(x)* I will apply the chain rule to indicate it's derivative.

Differentiating with respect to *x*:

$\frac{d}{dx}\left[x^3\right]=3x^2$

variables agree

Variables agree ⇒ use power rule

$\frac{d}{dx}\left[y^3\right]=3y^2\frac{dy}{dx}$

variables disagree

Variables disagree ⇒ use power rule and chain rule

$\frac{d}{dx}\left[2x^5+3y\right]=10x+3\frac{dy}{dx}$

variables disagree

variables agree

Practice: 1. $-3y^2 = x^4 + 5$ 2. $y^2 - 7y = \cos\left(x^3\right)$

Find $\frac{dy}{dx}$

Let's return to our 3 inverse functions and use implicit differentiation to find their derivatives:

$\frac{d}{dx}\left(\arctan(x)\right)$ $\frac{d}{dx}\left(\arcsin(x)\right)$ $\frac{d}{dx}\left(\ln(x)\right)$

Derivatives of some important Inverse functions (MEMORIZE THESE).

$\frac{d(\arctan x)}{dx} =$	$\frac{d(\arcsin x)}{dx} =$	$\frac{d(\ln x)}{dx} =$

Note: $\arctan x = \tan^{-1} x$ & $\arctan(\tan x) = \tan(\arctan x) = x$

$\arcsin x = \sin^{-1} x$ & $\arcsin(\sin x) = \sin(\arcsin x) = x$

Practice: Examples using the derivative rules we just found and applying rules we already learned:

a) $\frac{d(\arctan(t^2))}{dt}$ b) $\frac{d(\arcsin(\tan(\theta)))}{d\theta}$ c) $\frac{d\ln(x^2+1)}{dx}$

d) $\frac{d(t^2 \ln t)}{dt}$ 3) $\frac{d(\sqrt{1+\ln(2y)})}{dy}$ f) $\frac{d(\cos(\sin^{-1} x))}{dx}$

AB Calculus – Supplement
Derivative of the Inverse of a Function

Name: ______________________________

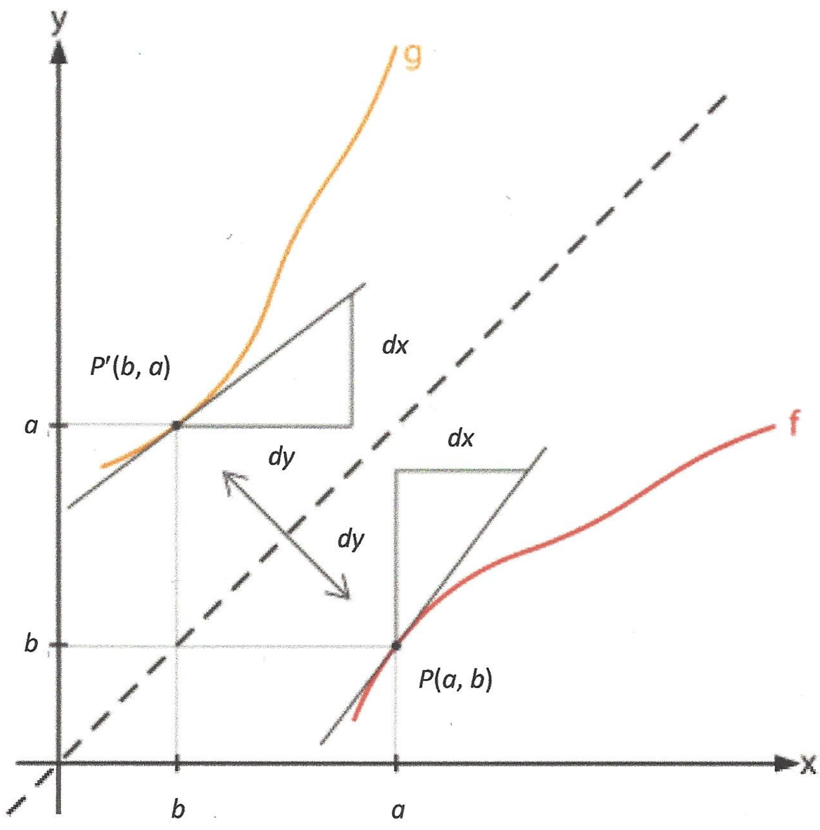
Class/Period: ________ Date: ____________

Suppose that f and g are inverse functions. What is the relationship between their derivatives?

- Algebraically: inverses are obtained by interchanging the x and the y coordinates and solving for y.
- Graphically: inverses are a reflection of the graph on the line $y = x$.

If f passes through the point (a, b), then the slope of the curve at $x = a$ is represented by $f'(a)$ and is represented by the ratio of the change in y over the change in x, $\dfrac{\Delta y}{\Delta x}$.

When this figure is reflected on the line $y = x$, we obtain the graph of the inverse f^{-1} and this passes through the point (b, a), with the horizontal and vertical sides of the slope triangle interchanged. So the slope of the line tangent to the graph of f^{-1} at $x = b$ is represented by the change in x over the change in y, $\dfrac{\Delta x}{\Delta y}$. This is the reciprocal of the slope of f at $x = a$.



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> Given (a, b) is a point on f, and g is the inverse of f,
>
> If $f'(a) = m$, then $g'(b) = \dfrac{1}{m}$.
>
> The derivative of the inverse of a function at a point is the reciprocal of the derivative of the function at the corresponding point.

Examples:

1) If $f(7) = 1$ and $f'(7) = 5$, and g is the inverse of f, then what is $g'(1)$?

2) Given $f(-2) = 5, f'(-2) = 6, f'(5) = -3$ and g is the inverse of f, what is $g'(5)$?

3) A function f and its derivative are shown on the table. If g is the inverse of f, find $g'(4)$ and $g'(-1)$.

x	$f(x)$	$f'(x)$
-3	4	0.25
2	-1	$-\frac{2}{3}$

4) Let $f(x) = \sqrt{x}$, and let g be the inverse function. Evaluate $g'(3)$.

5) If $f(2) = -3, f'(2) = \frac{3}{4}$, and g is the inverse of f, what is the equation of the tangent line to $g(x)$ and $x = -3$?

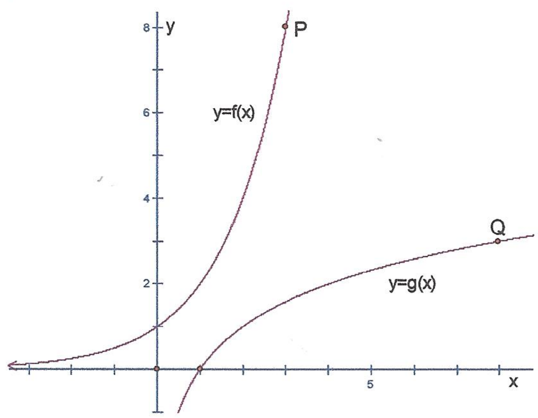
6) The following figure shows $f(x)$ and $f^{-1}(x)$. Using the given table, find:

a) $f(2), f^{-1}(2), f'(2), (f^{-1})'(2)$.

b) The equation of the tangent line at the points $P(3, 8)$ and $Q(8, 3)$.

c) What is the relationship between the two tangent lines?

x	$f(x)$	$f'(x) = g(x)$
0	1	0.7
1	2	1.4
2	4	2.8
3	8	5.5



7) Calculate $g'(1)$, where $g(x)$ is the inverse of the function $f(x) = x + e^x$ without solving for $g(x)$.

8) Calculate $g'(x)$, where $g(x)$ is the inverse of the function $f(x) = x^3 + 1$ without solving for $g(x)$.

9) Let $f(x) = \frac{1}{4}x^3 + x - 1$. Assume that $f(x)$ is one-to-one.

a. What is the value of $f^{-1}(x)$ when $x = 3$?

b. Find the slope of the tangent line to the curve $y = f^{-1}(x)$ at $x = 3$.

<u>Keys to Properly Solving Derivative of an Inverse Problems:</u>

- First, identify the point (a, b) on the function f using whatever information is given.
- Differentiate f.
- Take the reciprocal of the derivative of f. This is the derivative of f^{-1}.
- Evaluate the derivative of f^{-1} at the point (b, a).

Practice:

Given the following values for differentiable functions f and g.

x	f	f'	g	g'
1	2	½	-3	5
2	3	1	0	4
3	4	2	2	3
4	6	5	3	½

a. If $h(x) = f^{-1}(x)$, what is $h'(4)$?

b. If $h(x) = f^{-1}(x)$, what is $h'(2)$?

c. If $d(x) = g^{-1}(x)$, what is $d'(-3)$?

And these are not exactly on derivatives of inverses, but they are good practice nonetheless:

d. If $p(x) = g^2(x)$, what is $p'(3)$?

e. If $b = f \cdot g$ what is $b'(2)$?

f. If $n(x) = f(x^3)$, what is $n'(1)$?