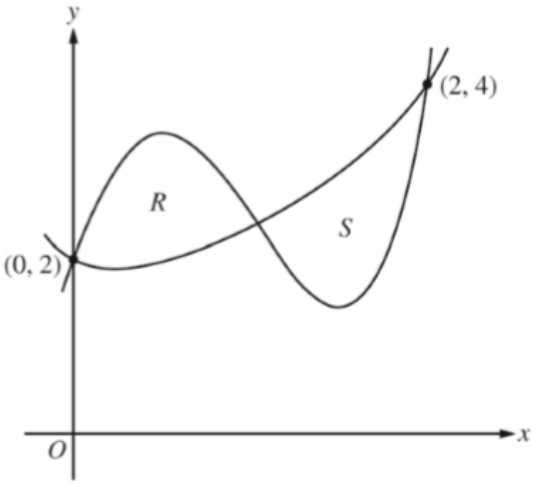
AP Calculus AB FRQ: Area and Volume Tuesday 3 April 2018 In Class Work: 4 FRQs Partner Collaborative

Show all work on a neatly separate sheet of paper. Your classmates will be grading your work using the AP scoring rubric

2015 #2 Calculator Active



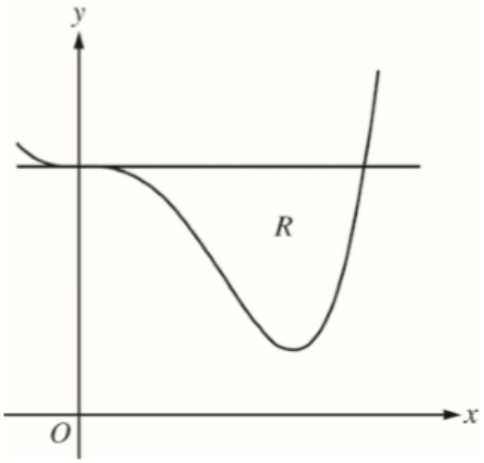
2. Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.

   (a) Find the sum of the areas of regions R and S.

   (b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

   (c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

2014 #2 Calculator Active



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

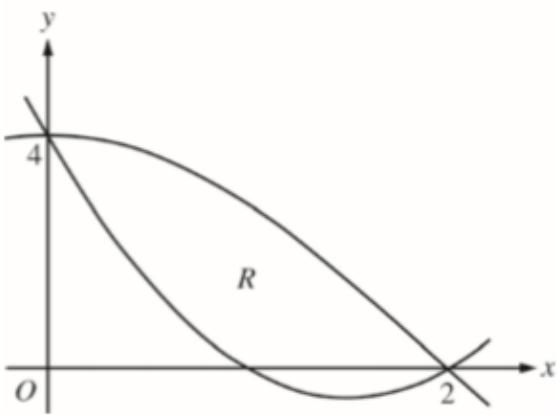
   (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

   (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

AP Calculus AB FRQ: Area and Volume Tuesday 3 April 2018 In Class Work: 4 FRQs Partner Collaborative

Show all work on a neatly separate sheet of paper. Your classmates will be grading your work using the AP scoring rubric

2013 #5 NON-Calculator



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

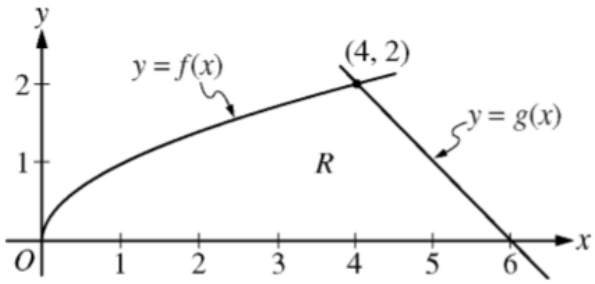
   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

2011 #3 NON-Calculator

No calculator is allowed for these problems.



3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

   (c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

Multiple Choice Questions & 1 FRQ on Area and Volume **HOMEWORK Tuesday 3 April, 2018**

Complete work for all questions on a separate sheet of paper.

1. (calculator not allowed)

   The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

   (A) 3π (B) $2\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

2. (calculator not allowed)

   What is the area of the region in the first quadrant bounded by the graph of $y = e^{\frac{x}{2}}$ and the line $x = 2$?

   (A) $2e-2$ (B) $2e$ (C) $\frac{e}{2}-1$ (D) $\frac{e-1}{2}$ (E) $e-1$

3. (calculator allowed)

   What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

   (A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

4. (calculator not allowed)

   The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

   (A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

5. (calculator not allowed)

   Let R be the region in the first quadrant bounded above by the graph of $y = \sqrt{x}$ and below by the graph of $y = x^2$. R is the base of a solid whose cross sections perpendicular to the $y-$axis are squares. Which of the following gives the volume of the solid?

   (A) $\int_0^1 \left(\sqrt{x} - x^2\right)^2 dx$ (B) $\int_0^1 \left(x - x^4\right) dx$ (C) $\int_0^1 \left(\sqrt{y} - y^2\right)^2 dy$ (D) $\int_0^1 \left(\sqrt{y} - y^2\right) dy$

6. (calculator allowed)

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$. R is the base of a solid whose cross sections perpendicular to the $x-$axis are semicircles. What is the volume of the solid?

(A) 29.815 (B) 174.268 (C) 348.537 (D) 443.771

7. (calculator not allowed)

The functions f and g are given by $f(x) = 2\sqrt{x}$ and $g(x) = x - 3$. Let R be the region bounded by the $x-$axis and the graphs of f and g. The graphs of f and g intersect in the first quadrant at the point $(9, 6)$. Which of the following gives the volume of the solid generated when R is revolved about the $x-$axis?

(A) $4\pi\int_0^3 x\,dx + \pi\int_3^9 \left(4x - (x-3)^2\right) dx$

(B) $4\pi\int_0^3 x\,dx + \pi\int_3^9 \left(2\sqrt{x} - (x-3)\right)^2 dx$

(C) $\pi\int_0^9 \left(2\sqrt{x} - (x-3)\right)^2 dx$

(D) $\pi\int_0^9 \left(4x - (x-3)^2\right) dx$

8. (calculator not allowed)

The function f is given by $f(x) = \ln x$. Which of the following limits is equal to the area between the graph of $f(x)$ and the $x-$axis from $x = 1$ to $x = 3$?

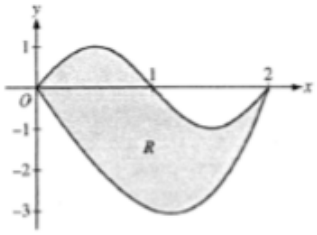
(A) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \ln\left(1 + \dfrac{2k}{n}\right)\dfrac{2}{n}$

(B) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \ln\left(1 + \dfrac{2k}{n} \cdot \dfrac{2}{n}\right)$

(C) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \ln\left(1 + \dfrac{2k}{n}\right)\dfrac{1}{n}$

(D) $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \ln\left(\dfrac{2k}{n}\right)\dfrac{2}{n}$

13. (calculator allowed)



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.