DAY

§4.6 Related Rates

Complete these Example Problems and HW problems in your notebook

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Example 2: Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?
(Answer: ft/min)

Example 3: A man 6 ft tall walks at a rate of 5 ft/sec toward a street light that is 16 ft above the ground. At what rate is the length of his shadow changing when he is 10 ft from the base of the light? **(Answer: –3 ft/sec)**

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Example 4: A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius, r, of the outer ripple is increasing at a constant rate of 1 ft/sec. When this radius is 4 ft, at what rate is the total area of the disturbed water increasing? **(Answer: 8π ft^2/sec)**

Example 5: Gravel is falling in a conical pile at the rate of 100 ft^3/min. Find the rate of change of the height of the pile when the height is 10 ft. Assume that the coarseness of the gravel is such that the radius of the cone is always equal to its height. **(Answer: $\frac{1}{\pi} \approx 0.318$ ft/min)**

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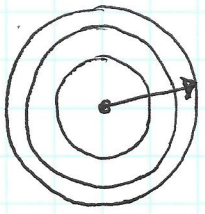
#4)



$\frac{dr}{dt} = 1 \frac{ft}{sec}$ $r = 4$ ft. $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

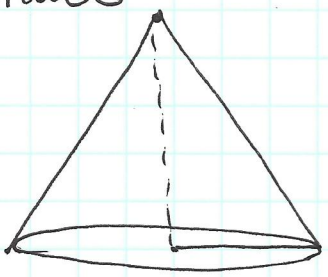
$\frac{dA}{dt} = 2\pi(4)(1)$

$\frac{dA}{dt} = 8\pi \frac{ft^2}{sec}$

$\frac{dr}{dt}$ is positive b/c Area of circle is increasing.

ATQ. The area of the disturbed water is increasing at a rate of $8\pi \frac{ft^2}{sec}$ when the radius is 4ft.

#5) Gravel



$\frac{dV}{dt} = 100 \frac{ft^3}{min}$ Find $\frac{dh}{dt} = ? \frac{ft}{min}$

& $r = h$ when $h = 10$ ft.

$\therefore \frac{dr}{dt} = \frac{dh}{dt}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{\pi}{3}\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{3}\left(2h^2\frac{dh}{dt} + h^2\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{3}(3h^2)\frac{dh}{dt}$

$\frac{dV}{dt} = \pi(h^2)\frac{dh}{dt}$

$\frac{dh}{dt} = \left(\frac{1}{\pi h^2}\right)\left(\frac{dV}{dt}\right) = \left(\frac{1}{\pi 10^2}\right)(100) = \frac{1}{\pi} \frac{ft}{min}$

The height of the gravel in the conical pile is increasing at a rate of $\frac{1}{\pi} \approx 0.3183 \frac{ft}{min}$ at the moment the height is 10ft.