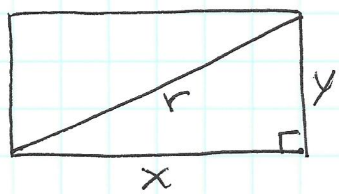
p.237-242 #17, 25, 27, 29

§4.6 HW DAY 72

(17)



$x = 8\text{cm}$ $\quad y = 6\text{cm}$

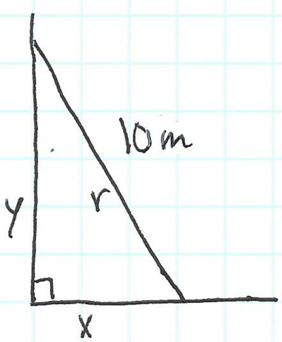
$\frac{dx}{dt} = 0 \frac{\text{cm}}{\text{sec}}$ $\quad \frac{dy}{dt} = 3 \frac{\text{cm}}{\text{sec}}$

$$x^2 + y^2 = r^2$$
$$8^2 + 6^2 = r^2$$
$$10 = r$$

ATQ. The length of the diagonal is increasing at a rate of $\frac{9}{5} = 1.8 \frac{\text{cm}}{\text{sec}}$ at the moment $y = 6\text{cm}$.

$$x^2 + y^2 = r^2$$
$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2r \frac{dr}{dt}$$
$$x \frac{dx}{dt} + y \frac{dy}{dt} = r \frac{dr}{dt}$$
$$\left(\frac{1}{r}\right)\left(x \frac{dx}{dt} + y \frac{dy}{dt}\right) = \frac{dr}{dt}$$
$$\left(\frac{1}{10}\right)\left(8(0) + 6(3)\right) = \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{9}{5} \frac{\text{cm}}{\text{sec}}$$
$$\frac{dr}{dt} = 1.8 \text{ cm/sec}.$$

(25)



$\frac{dx}{dt} = +\frac{1}{2} \frac{\text{m}}{\text{sec}}$

$\frac{dy}{dt} = ? \frac{\text{m}}{\text{sec}}$ will be negative.

a) $x = 4\text{ft}$

$$x^2 + y^2 = r^2$$
$$y = \sqrt{10^2 - 4^2}$$
$$y = \sqrt{2^2(5^2 - 2^2)}$$
$$y = 2\sqrt{21}$$

$$x^2 + y^2 = r^2$$
$$x \frac{dx}{dt} + y \frac{dy}{dt} = r \frac{dr}{dt}$$
$$\frac{dy}{dt} = \left(r \frac{dr}{dt} - x \frac{dx}{dt}\right)\left(\frac{1}{y}\right)$$
$$\frac{dy}{dt} = \left((10)(0) - (4)\left(\tfrac{1}{2}\right)\right)\left(\frac{1}{2\sqrt{21}}\right)$$
$$\frac{dy}{dt} = -2\left(\frac{1}{2\sqrt{21}}\right)$$
$$\frac{dy}{dt} = \frac{-1}{\sqrt{21}} \frac{\text{m}}{\text{sec}}.$$

a) When $x = 4\text{ft}$ the ladder is sliding away from the base of the wall at a rate of $\frac{1}{\sqrt{21}} \frac{\text{m}}{\text{sec}}$

$$x^2 + y^2 = r^2$$
$$y = \sqrt{10^2 - 8^2}$$
$$y = 6$$

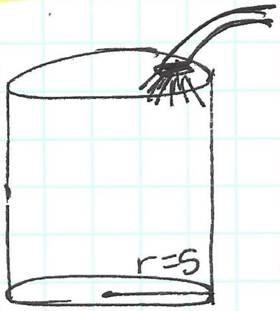
b) When $x = 8\text{ft}$ the ladder is sliding away from the wall at a rate of $\frac{2}{3} \frac{\text{m}}{\text{sec}}$.

b)
$$\frac{dy}{dt} = \left(-8\left(\tfrac{1}{2}\right)\right)\left(\frac{1}{6}\right)$$
$$\frac{dy}{dt} = \frac{-4}{6}$$
$$\frac{dy}{dt} = \frac{-2}{3} \frac{\text{m}}{\text{sec}}$$

§4.6 DAY 72 HW p. 237-242 #27, 29.

(27)

h = 20 m



$\frac{dr}{dt} = 0$

$\frac{dV}{dt} = 3 \frac{m^3}{min}$

when cylinder is ½ full
h = 10 m.

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi \left(2rh \frac{dr}{dt} + r^2 \frac{dh}{dt}\right)$

$\frac{dV}{dt} = \pi \left(2rh(0) + r^2 \frac{dh}{dt}\right)$

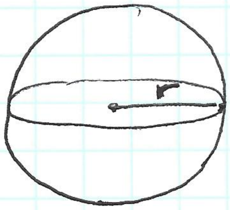
$\left(\frac{dV}{dt}\right)\left(\frac{1}{\pi r^2}\right) = \frac{dh}{dt}$

$(3)\frac{1}{\pi 5^2} = \frac{3}{25\pi} \frac{m}{min}$

$0.03189 \frac{m}{min}$

ATQ: The water level (height) is rising at a rate of $\frac{3}{25\pi} \frac{m}{min} \approx 0.03189 \frac{m}{min}$ when the cylinder is half full.

(29)



$\frac{dr}{dt} = 2 \frac{cm}{sec}$

when r = 10 cm at what rate is air being pumped into balloon?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (10)^2 (2)$

$= 4\pi \, 200$

$\frac{dV}{dt} = 800\pi \frac{cm^3}{sec}$

ATQ: When the radius of the balloon is 10 cm, the volume of the balloon is increasing at a rate of $800\pi \frac{cm^3}{sec}$

Volume rate = air pumped into balloon rate.