DAY

§4.6 Related Rates

Complete these Example Problems and HW problems in your notebook

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Example 2: Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?
(Answer: ft/min)

Example 3: A man 6 ft tall walks at a rate of 5 ft/sec toward a street light that is 16 ft above the ground. At what rate is the length of his shadow changing when he is 10 ft from the base of the light? **(Answer: –3 ft/sec)**

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Example 4: A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius, *r*, of the outer ripple is increasing at a constant rate of 1 ft/sec. When this radius is 4 ft, at what rate is the total area of the disturbed water increasing? **(Answer: 8π ft^2/sec)**

Example 5: Gravel is falling in a conical pile at the rate of 100 ft^3/min. Find the rate of change of the height of the pile when the height is 10 ft. Assume that the coarseness of the gravel is such that the radius of the cone is always equal to its height. **(Answer: $\frac{1}{\pi} \approx 0.318$ ft/min)**

HW: Related Rates DAY 71-72 HW

HW #1: The width of a rectangle is increasing at a rate of 2 cm/sec and its length is increasing at a rate of 3 cm/sec. At what rate is the area of the rectangle increasing when its width is 4 cm and length is 5 cm? At what rate is the length of the diagonal of the rectangle increasing? At what rate is the perimeter of the rectangle increasing?

HW #2: A spherical ball 8 inches in diameter is coated with a layer of ice of uniform thickness. If the ice melts at a rate of 10 in^3/min, how fast is the thickness of the ice decreasing when it is 2 inches thick? How fast is the outer surface area of the ice decreasing at this time?

HW #3: A baseball diamond is a square 90 feet on one side. A runner travels from home plate to first base at 20 ft/sec. How fast is the runner's distance from second base changing when the runner is halfway to first base?

HW #4: A rocket rises vertically from a point on the ground that is 100 m from an observer at ground level. The observer notes that the angle of elevation is increasing at a rate of 12 degrees per second when the angle of elevation is 60 degrees. Find the speed of the rocket at that instant. *convert to radians

*convert to radians

HW #5: If $x^2 + y^2 = 25$ and $\frac{dy}{dt} = 6$ cm/sec, find $\frac{dx}{dt}$ when $y = 4$ cm.

HW #6: Sand is being dumped on a pile in such a way that it always forms a cone whose radius equals its height. If the sand is being dumped at a rate of 10 ft^3/min, at what rate is the height of the pile increasing when there is 1000 ft^3 of sand on the pile?

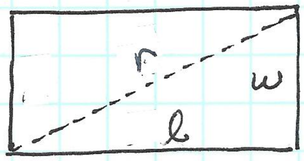
HW 4.6 RELATED RATES p.20 # 1-6

p.20

#1)

A) $\frac{dw}{dt} = 2\ \frac{cm}{sec}$

$\frac{dl}{dt} = 3\ \frac{cm}{sec}$



$A = l \cdot w$

$\frac{dA}{dt} = \frac{dl}{dt} w + l \cdot \frac{dw}{dt}$

$\frac{dA}{dt} = (3)(4) + (5)(2)$

$\frac{dA}{dt} = 22\ \frac{cm^2}{sec.}$

ATQ: when $w = 4cm$, $l = 5cm$ the area of the rectangle is increasing at a rate of 22 cm^2/sec.

B) $\frac{dr}{dt} = ?$

$4^2 + 5^2 = r^2$

$\sqrt{41} = r$

$l^2 + w^2 = r^2$

$\cancel{2} l \frac{dl}{dt} + \cancel{2} w \frac{dw}{dt} = \cancel{2} r \frac{dr}{dt}$

$\frac{(5)(3) + (4)(2)}{\sqrt{41}} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{23}{\sqrt{41}} \approx 3.591$ or 3.592

ATQ: When $w = 4cm$ & $l = 5cm$ the length of the diagonal is increasing at a rate of 3.951 cm/sec.

c) $\frac{dP}{dt} = ?$

$P = 2(l + w)$

$\frac{dP}{dt} = 2\left(\frac{dl}{dt} + \frac{dw}{dt}\right)$

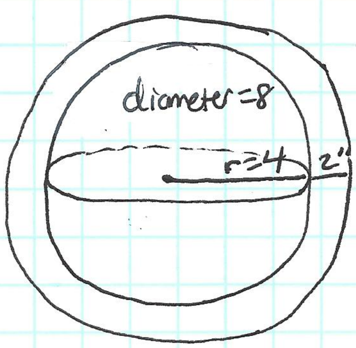
$\frac{dP}{dt} = 2(3 + 2)$

$\frac{dP}{dt} = 10\ \frac{cm}{sec.}$

ATQ: When $w = 4cm$ & $l = 5cm$ the Perimeter is increasing at a rate of 10 cm/sec.

#2)

A)



$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = -10\ \frac{in^3}{min}$

$V = \frac{4\pi}{3} r^3$

when 2" thick

$\therefore r = 6$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi r^2}$

$\frac{dr}{dt} = \frac{-10}{4\pi(6)^2} = \frac{-5}{72\pi}\ \frac{in}{min}$

ATQ: When the ice is 2 inches thick, the thickness of the ice is decreasing at a rate of $\frac{-5}{72\pi}$ in/min

B) $SA = 4\pi r^2$

$\frac{d(SA)}{dt} = 8\pi r \frac{dr}{dt}$

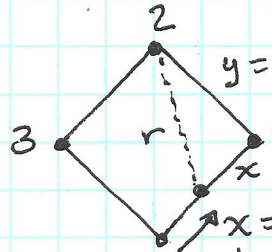
$= 8\pi(6)\left(\frac{-5}{72\pi}\right) = \frac{\cancel{4} \cdot 2 \cdot \cancel{6}(-5)\cancel{\pi}}{\cancel{4} \cdot 3 \cdot \cancel{6} \cdot \cancel{\pi}} = \frac{-10}{3}\ \frac{in}{min}$

ATQ: when the ice is 2" thick the surface area is decreasing at a rate of $\frac{10}{3}\ \frac{in}{min}$.

H.W. 4.6 RELATED RATES p.20 #3,4,5

p.20

#3)



$y = 90 \quad \frac{dy}{dt} = 0 \frac{ft}{sec}$

$x = 45 ft.$

$\frac{dx}{dt} = -20 \frac{ft}{sec}.$

$45^2 + 90^2 = r^2$

$45^2(1+2^2) = r^2$

$45\sqrt{5} = r$

$x^2 + y^2 = r^2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = r\frac{dr}{dt}$

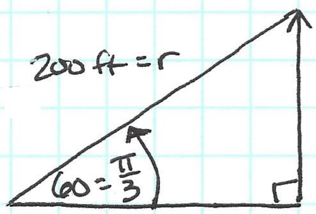
$\frac{dr}{dt} = \left(\frac{1}{r}\right)\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$

$\frac{dr}{dt} = \frac{1}{45\sqrt{5}}\left(45\cdot(-20) + 0\right)$

$\frac{dr}{dt} = \frac{-20}{\sqrt{5}} = -4\sqrt{5} \frac{ft}{sec}.$

ATQ: When the runner is ½ way to 1st base the distance the runner is from 2nd base is decreasing at a rate of $4\sqrt{5}$ ft/sec.

#4)



$y = 100\sqrt{3} ft$

$\frac{dy}{dt} = ?$

$x = 100 ft$

$\frac{dx}{dt} = 0 \frac{ft}{sec}$

$\frac{d\theta}{dt} = \frac{\pi}{15} \frac{rad}{sec}.$

$\tan\theta = \frac{y}{100}$

$100 \tan\theta = y$

$100 \sec^2\theta \frac{d\theta}{dt} = \frac{dy}{dt}$

$100\left(\frac{2}{1}\right)\left(\frac{\pi}{15}\right) = \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{40\pi}{3} \frac{ft}{sec}.$

$\theta = \frac{\pi}{3} \quad \cos\theta = \frac{1}{2} \; \& \; \sec\theta = \frac{2}{1}$

USE RADIANS B/C Radians are a linear unit. If radius is in feet then $\frac{radians}{sec} = \frac{feet}{sec}$. Rocket's height is increasing at a rate of $\frac{40\pi}{3} \frac{ft}{sec} \approx 42 \frac{ft}{sec}.$

#5) CIRCLE w/ radius 5

$x^2 + y^2 = 25$

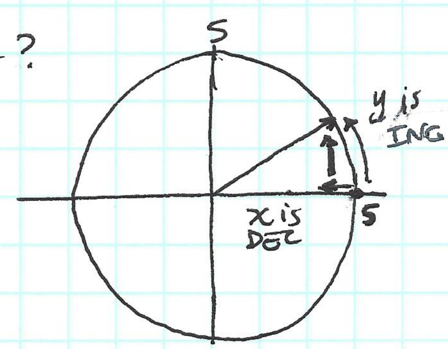
when $y = 4$ cm & $\frac{dy}{dt} = 6 \frac{cm}{sec} \Rightarrow x = 3$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$

$\frac{dx}{dt} = ?$

$\frac{dx}{dt} = \left(-y\frac{dy}{dt}\right)\left(\frac{1}{x}\right)$

$\frac{dx}{dt} = -4(6)\left(\frac{1}{3}\right) = -8 \frac{cm}{sec}$

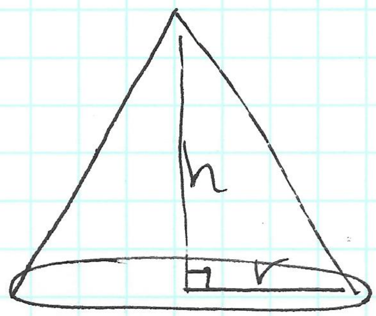


When y = 4 cm x is decreasing at a rate of $8 \frac{cm}{sec}$

HW 4.6 RELATED RATES p.20 ~~#6~~

p.20

#6



$$V=\frac{1}{3}\pi r^2 h \quad \& \quad h=r \quad \& \quad \frac{dV}{dt}=10\frac{ft^3}{min}$$

$$\frac{dh}{dt}=\frac{dr}{dt}$$

$$V=\frac{\pi}{3}h^3$$

$$\frac{dV}{dt}=\pi h^2\frac{dh}{dt}$$

$$V=1000\ ft^3$$

$$\frac{\pi}{3}h^3=1000$$

$$h^3=\frac{3000}{\pi}$$

$$h=\sqrt[3]{\frac{3000}{\pi}}\ ft.$$

$$\frac{dh}{dt}=\frac{\frac{dV}{dt}}{(\pi h^2)}$$

$$\pi\left(\frac{1}{\pi^{2/3}}\right)=\sqrt[3]{\pi}$$

$$\frac{dh}{dt}=\frac{+10}{\pi\left(\sqrt[3]{\frac{3000}{\pi}}\right)^2}\ \frac{ft^3/min}{ft^2}$$

$$\frac{dh}{dt}=0.0328\ \frac{ft}{min} \qquad \frac{10}{\sqrt[3]{\pi}\ \sqrt[3]{3000^2}}\ \frac{ft}{min}$$

ATQ: When there is 1000 ft^3 of sand on the pile, the height is increasing at a rate of 0.0328 ft/min.