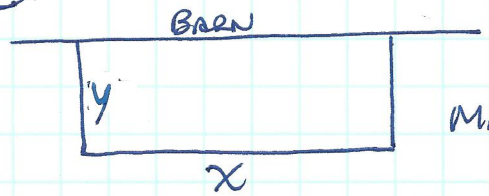
$\boxed{\text{DAY 70}}$ NOTES ANSWERS p.17 #7, 8, 9, 10, 12

(7) Perimeter $= 100\text{ ft} = x + 2y \longrightarrow y = \dfrac{100-x}{2}$

Domain $x > 0$



$\text{Area} = x \cdot y = x\left(\dfrac{100-x}{2}\right)$

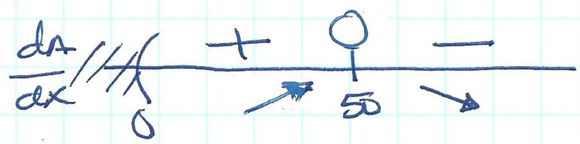
MAXIMIZE AREA

$= \frac{1}{2}(x)\left(50 - \frac{1}{2}x\right)$

$$\frac{dA}{dx} = \left(\tfrac{1}{2}\right)\left(50 - \tfrac{1}{2}x\right) + \tfrac{1}{2}x\left(-\tfrac{1}{2}\right)$$

$$\frac{dA}{dx} = 25 - \tfrac{1}{4}x - \tfrac{1}{4}x = 25 - \tfrac{1}{2}x \qquad \boxed{\frac{d^2A}{dx^2} = -\frac{1}{2}}$$

$\dfrac{dA}{dx} = 0 \qquad x = 50$



$\boxed{\text{JUSTIFY}} \rightarrow$ Area is maximized when $x = 50$ ft b/c $\dfrac{d^2A}{dx^2} < 0$ so Area is concave down and has MAX at $x = 50$. } 2nd Derivative Test.

ATQ. $x = 50\text{ ft}$ $y = 25\text{ ft}$ Area $= 1250\text{ ft}^2$

(8) Product $(x)(y) = 192 \rightarrow y = \dfrac{192}{x}$

domain $x > 0$ $y > 0$

MIN Sum $= 3x + y$

$$\text{Sum} = 3x + \frac{192}{x}$$

$$\frac{dS}{dx} = 3 - \frac{192}{x^2} \longrightarrow \frac{d^2S}{dx^2} = \frac{2(192)}{x^3} = \frac{384}{x^3}$$

$$= \frac{3x^2 - 192}{x^2}$$

$$= \frac{3(x^2 - 64)}{x^2}$$

$$= \frac{3(x-8)(x+8)}{x^2}$$

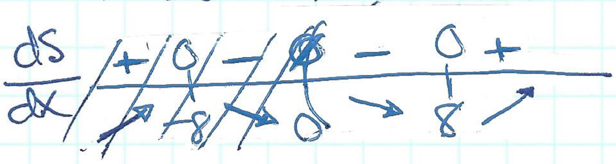
$\left.\dfrac{d^2S}{dx^2}\right|_{x=8} > 0 \quad \therefore$ Sum is concave up & $S(8)$ is a minimum by 2nd Deriv Test

ATQ: $x = 8$
$y = 24$
Sum $= 48$.

$\dfrac{dS}{dx} = 0 \qquad \dfrac{dS}{dx}$ und

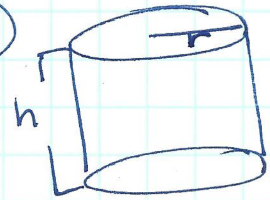
Domain $x > 0$

$x = \pm 8 \qquad x = 0 \quad \therefore$ exclude $x = 0$, $x = -8$



DAY 70 p. 17 #9, 10, 12 NOTES ANSWERS.

(9)



$$V = \underset{(12)(1.80469)}{21.65628 \text{ in}^3} = \pi r^2 h$$

$$h = \frac{(12)(1.08469)}{\pi r^2}$$

MINIMIZE SA.

$$SA = 2\pi r h + 2\pi r^2$$

$$SA = 24\pi r\left(\frac{1.80469}{\pi r^2}\right) + 2\pi r^2$$

* Can is 12 fluid ounces.

$$SA = \frac{43.31256}{r} + 2\pi r^2$$

$$\frac{d(SA)}{dr} = -\frac{43.31256}{r^2} + 4\pi r$$

$$\frac{d(SA)}{dr} = \frac{-43.31256 + 4\pi r^3}{r^2}$$

$$\frac{d(SA)}{dr} = 0 \qquad r^3 = \frac{43.31256}{4\pi}$$

$$r = \sqrt[3]{\frac{43.31256}{4\pi}}$$

$$r \approx 1.510548463$$

$$\frac{d^2(SA)}{dr^2} = \frac{86.62512}{r^3} + 4\pi$$

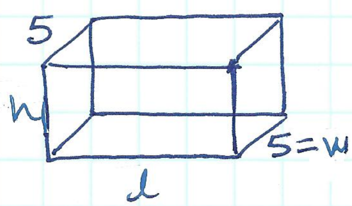
$$\left.\frac{d^2(SA)}{dr^2}\right|_{r=1.5105} = 37.699 > 0$$

∴ SA is minimized when $r = 1.5105$ b/c SA is concave up at this critical point for r. by <u>2nd Derivative Test</u>.

(10) Glass Fish Tank. $72 \text{ ft}^3 = V = l \cdot w \cdot h$

$$V = 5lh = 72$$

$$h = \frac{72}{5l}$$



Domain: $h > 0,\ l > 0$

MINIMIZE COST!

$$\$10(5l) + \$5(2l+10)(h) = \text{Cost}.$$

$$50l + 10h(l+5) = \text{Cost}.$$

$$\text{Cost} = 50l + 10\left(\frac{72}{5l}\right)(l+5)$$

$$\text{Cost} = 50l + \left(\frac{144}{l}\right)(l+5)$$

$$\text{Cost} = 50l + 144 + \frac{720}{l}$$

$$\boxed{\text{JUSTIFY}}\ \left.\frac{d^2C}{dl^2}\right|_{l=\sqrt{\frac{72}{5}}} > 0 \ ∴$$

Cost function is concave up at critical point $l = \sqrt{\frac{72}{5}}$ ∴ Minimizing cost by 2nd Der. Test.

ATQ Cost = \$523.47

$l = 3.7947$ ft & $h = 3.7947$ ft

$$\frac{dC}{dl} = 50 - \frac{720}{l^2} = \frac{50l^2 - 720}{l^2}$$

$$\frac{dC}{dl} = 0 \qquad l = \pm\sqrt{\frac{720}{50}} = \pm\sqrt{\frac{72}{5}}$$

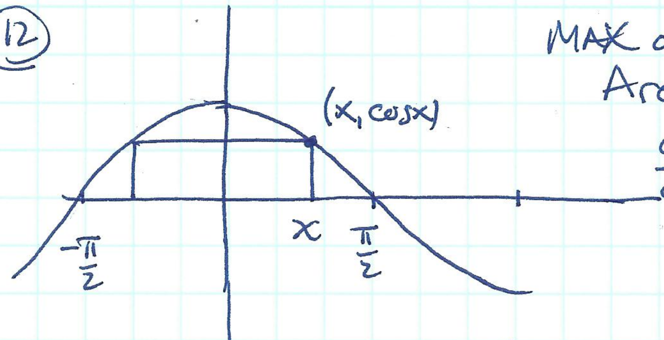
$\frac{dC}{dl}$ und $\quad l = 0$

$\frac{dC}{dl}$: $+$ 0 $-$ Ø $-$ 0 $+$ at $-\sqrt{\frac{72}{5}}$, 0, $\sqrt{\frac{72}{5}}$

$$\left.\frac{d^2C}{dl^2} = \frac{+1440}{l^3}\right|_{l=\sqrt{\frac{72}{5}}} = 26.352 > 0$$

DAY 70 p.17 #12 NOTES ANSWERS

(12)



domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$

MAX of rectangle.

Area $= (2x)(\cos x)$

$\frac{dA}{dx} = 2\cos x - 2x\sin x$

$\underbrace{2(\cos x - x\sin x) = 0}_{\text{Solve on graphing calculator.}}$

$x = \pm 0.86033359$

$\frac{dA}{dx}$: — 0 + 0 —

$\searrow$ -0.860 MIN. $\nearrow$ $+0.860$ MAX $\searrow$

JUSTIFY

$\frac{dA}{dx}$ changes signs (+) to (−)

at $x = 0.860$ verifying

a maximum area when $x = 0.860$ & height $= 0.652$ & Area $= 1.122$