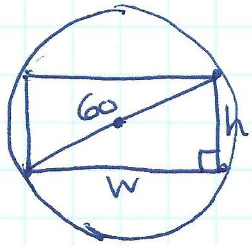
HW §4.3 $\boxed{\text{DAY 70}}$ p. 210-215 #17, 24, 25, 28, 42

(17) Strength of Beam $= kwh^2$ & assume proportionality constant $k=1$

$= wh^2$



$w^2+h^2=60^2$

$h^2=60^2-W^2$

Strength $= w(60^2-w^2)$

$\frac{dS}{dW} = (60^2-w^2) + w(-2w)$

$\frac{dS}{dw} = 60^2 - 3w^2$

domain: $h>0$ cm

$w>0$ cm

$\frac{dS}{dw}=0$ $\quad w^2 = \frac{60^2}{3} = 1200$

$w = \pm\sqrt{1200} = \pm 20\sqrt{3}$

$\boxed{\text{JUSTIFY}}$:

$\frac{d^2S}{dw^2} = -6w \Big|_{+20\sqrt{3}} = -120\sqrt{3} < 0$

ATQ: $h = 60^2 - \frac{60^2}{3} = \frac{2}{3}(60^2)$

$= 2400$ cm

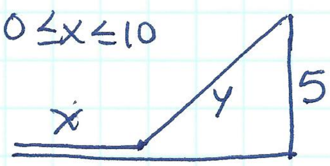
$w = 20\sqrt{3}$ cm

Strength $= w \cdot h^2$

$= 199{,}532{,}253$ cm^3

∴ Strength function is concave down at critical point $w=20\sqrt{3}$ which maximizes the strength of the beam. by 2nd Deriv. Test.

(24) $0 \leq x \leq 10$

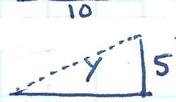


(25)

min $x=0$

max $x=10$

$\rightarrow$ min $y=5$

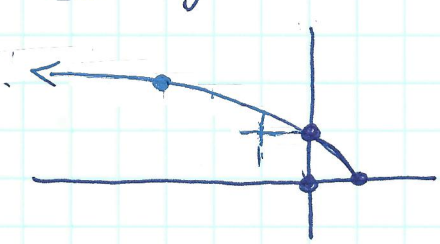


$\rightarrow$ max $y = \sqrt{10^2+5^2}$

max $y = \sqrt{5^2(2^2+1^2)}$

max $y = 5\sqrt{5}$

(28) $y=\sqrt{1-x}$ find (x,y) point closest to $(0,0)$



$d^2 = (x-0)^2+(y-0)^2$

$d^2 = x^2+(\sqrt{1-x})^2$

$d^2 = x^2+(1-x)$

$d^2 = x^2-x+1$

$\frac{d(d^2)}{dx} = 2x-1 = 0$

$x = \frac{1}{2}$

$y = \sqrt{1-\frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

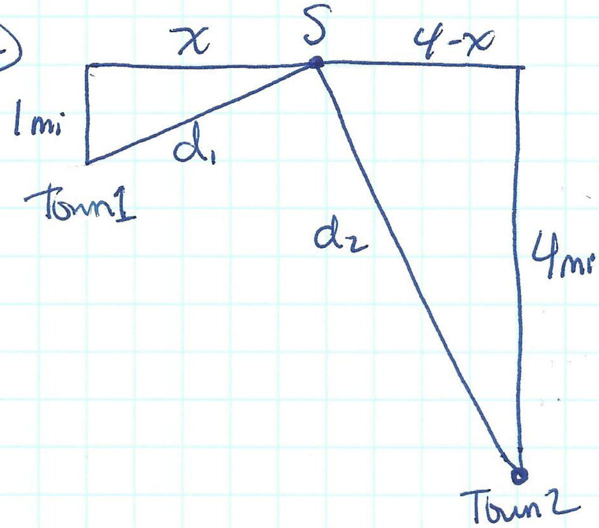
$\boxed{\text{JUSTIFY}}$

$\frac{d^2(d^2)}{dx^2} = 2 > 0$

∴ d^2 function is concave up at critical point $x=\frac{1}{2}$ where distance from $(0,0)$ to $(\frac{1}{2}, \frac{\sqrt{2}}{2})$ is minimized.

HW §4.3 $\boxed{\text{DAY 70}}$ p. 210-215 #42

42



Minimize total distance d_1+d_2

$$D^2=(1^2+x^2)+((4-x)^2+4^2)$$

$$D^2=(1+x^2+16-8x+x^2+16)$$

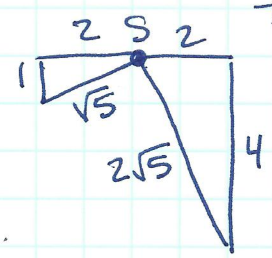
$$D^2=2x^2-8x+33$$

$$\frac{d(D^2)}{dx}=4x-8$$

$$\frac{d(D^2)}{dx}=0 \qquad x=2$$

$x=2 \rightarrow d_1=\sqrt{5}$

$4-x=2 \rightarrow d_2=2\sqrt{5}$



total distance of pipe = $3\sqrt{5}$ miles.
6.70820 miles.

$$\frac{d^2(D^2)}{dx^2}=4>0$$ ∴ D^2 function is concave up at critical point $x=2$.

∴ Distance^2 is minimized when $x=2$ by 2nd Deriv Test.