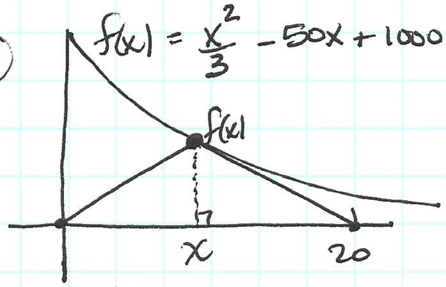
§4.3 $\boxed{\text{DAY 69}}$ HW p. 210–215 #11, 14, 20, 23

(11) $f(x) = \frac{x^2}{3} - 50x + 1000$



base = 20

ht = $f(x)$

$A = \frac{1}{2}(20)(f(x))$

$A = 10\, f(x)$

MAX $A_\Delta = \frac{1}{2}(20)(f(x))$

$A = 10\left(\frac{x^2}{3} - 50x + 1000\right)$

$\frac{dA}{dx} = 10\left(\frac{2}{3}x - 50\right)$

$\frac{dA}{dx} = 20\left(\frac{x}{3} - 25\right)$

$\frac{dA}{dx} = \frac{20}{3}(x - 75) = 0 \quad x = 75.$

$\frac{dA}{dx}$ [−] — 0 +

0 ↘ 20 ↘ 75 ↗

$x = 75$ not within the domain.

and it minimizes area but we want to maximize area.

☆ So look at endpoints.

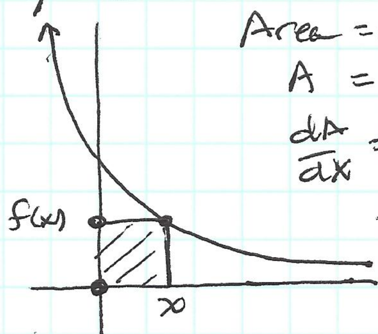
$x = 0 \quad A(0) = 10{,}000$ ← ∴ $x = 0$ maximizes area

$x = 20 \quad A(20) = 1333.\frac{1}{3}$

$\frac{dA}{dx} < 0$ on $[0, 20]$

So choose $x = 0$ to maximize area.

(14) $y = e^{-2x}$



a) MAX Area

Area $= (x)(f(x))$

$A = x e^{-2x}$

$\frac{dA}{dx} = e^{-2x} + x(-2e^{-2x})$

$= e^{-2x}(1 - 2x) = 0$

$x = \frac{1}{2}$

$\frac{dA}{dx}$ + 0 −

↗ $\frac{1}{2}$ ↘

$\frac{dA}{dx}$ changes signs ⊕ to ⊖ at $x = \frac{1}{2}$

∴ Area is maximized when $x = \frac{1}{2}$

Area $= \left(\frac{1}{2}\right) \cdot f\left(\frac{1}{2}\right)$

$A = \left(\frac{1}{2}\right) e^{-1}$

$A = \frac{1}{2e} \approx 0.18393$

b) MIN perimeter

Perim $= 2x + 2f(x)$

$P = 2x + 2e^{-2x}$

$\frac{dP}{dx} = 2 - 4e^{-2x} = 0$

$4e^{-2x} = 2$

$e^{-2x} = \frac{1}{2}$

$-\ln 2 = -2x$

$\frac{1}{2}\ln 2 = x$

$\ln\sqrt{2} = x$

$\frac{dP}{dx}$ − 0 +

↘ $\ln\sqrt{2}$ ↗

.3465

$\frac{dP}{dx}$ changes sign ⊖ to ⊕ at $x = \ln\sqrt{2}$

∴ Perim is minimized at $x = \ln\sqrt{2}$.

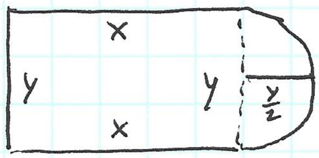
$P = 2\left(\frac{1}{2}\right) + 2f\left(\frac{1}{2}\right)$

$P = 1 + 2e^{-1}$

$P = 1 + \frac{2}{e} = \frac{e+2}{e} \approx 1.73575$

§4.3 $\boxed{\text{DAY 69}}$ HW p.210 #20,23.

20



$\frac{1}{2}\left(\frac{\cancel{2}\pi r}{\cancel{2}}\right)$

a) $A = x\cdot y + \frac{\pi}{2}\left(\frac{y}{2}\right)^2$

$A = x\cdot y + \frac{\pi}{8}y$

b) $P = 2x + y + \frac{1}{2}\left(2\pi \frac{y}{2}\right)$

$P = 2x + y + \frac{\pi}{2}y$

$2x + \left(1+\frac{\pi}{2}\right)y$

c) MAX. AREA when P = 100 $\longrightarrow$ $100 = 2x + y\left(\frac{2+\pi}{2}\right)$

$\frac{100-2x}{\frac{2+\pi}{2}} = y$

$y = \frac{4(50-x)}{(2+\pi)}$

$A = x\left(\frac{4(50-x)}{2+\pi}\right) + \frac{\pi}{8}\left(\frac{4(50-x)}{2+\pi}\right)$

$A = \left(\frac{4}{2+\pi}\right)(x)(50-x) + \left(\frac{-\pi}{4+2\pi}\right)(50-x)$

$A = \left(\frac{4}{2+\pi}\right)(50x-x^2) + \left(\frac{-\pi}{4+2\pi}\right)(50-x)$

$\frac{dA}{dx} = \left(\frac{4}{2+\pi}\right)(50-2x) + \left(\frac{-\pi}{4+2\pi}\right)(-1)$

$\left(\frac{4(50)}{2+\pi} - \frac{\pi}{4+2\pi}\right) - \left(\frac{8}{2+\pi}\right)x$

$\frac{dA}{dx} = \frac{400-\pi}{4+2\pi} - \frac{8}{2+\pi}x = 0$

$\left(\frac{400-\pi}{2\cancel{(2+\pi)}}\right)\left(\frac{\cancel{2+\pi}}{8}\right) = x$

$\frac{400-\pi}{16} = x$

$\frac{400-\pi}{16} = x \approx 24.80365$

$\frac{dA}{dx}$: + 0 − at $\frac{400-\pi}{16}$, $\nearrow$ $\searrow$

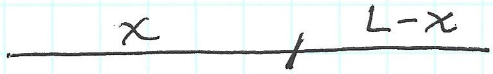
Area is maximize when $x = \frac{400-\pi}{16}$

b/c $\frac{dA}{dx}$ changes signs + to −.

§ 4.3 $\boxed{\text{DAY 69}}$ p. 211 #23

23

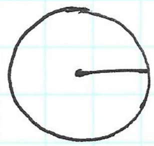
L cm of wire

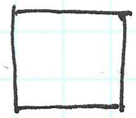


$2\pi r = C$

$2\pi r = x$

$r = \dfrac{x}{2\pi}$





$4s = P$

$4s = L - x$

$s = \dfrac{L-x}{4}$

$$\text{Area}_{\odot} = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi} = \frac{1}{4\pi}x^2$$

$$A_{\square} = s^2 = \left(\frac{L-x}{4}\right)^2 = \frac{1}{16}(L-x)^2$$

Goal MIN & MAX AREA.

$$\text{Total Area} = \frac{1}{4\pi}x^2 + \frac{1}{16}(L-x)^2$$

a) MINIMIZE AREA

$$\frac{dA}{dx} = \frac{1}{2\pi}x - \frac{1}{8}(L-x)$$

$$= \frac{1}{2\pi}x - \frac{L}{8} + \frac{1}{8}x$$

$$= \left(\frac{1}{2\pi} + \frac{1}{8}\right)x - \frac{L}{8} = 0$$

Positive Linear expression $\left\{ +\left(\dfrac{8+2\pi}{16\pi}\right)x - \dfrac{L}{8} = 0 \right.$

$$x = \left(\frac{L}{8}\right)\left(\frac{16\pi}{8+2\pi}\right)$$

$$x = \frac{(L)(2\pi)}{(8+2\pi)}$$

$\dfrac{dA}{dx}$: − 0 + at $\dfrac{L(2\pi)}{(8+2\pi)}$ ↘ ↗

Area will be minimized when $x = \dfrac{L(2\pi)}{(8+2\pi)}$ b/c $\dfrac{dA}{dx}$ changes signs − to +

b) MAXIMIZE AREA

How to maximize area? Look at the endpoints of the domain

Either $x = 0$ or $x = L$

If $x = 0$ then only make a square & don't make a circle & $A = \dfrac{L^2}{16}$

If $x = L$ then only make a circle & don't make a square

$r = \dfrac{L}{2\pi}$ $\quad A = \pi\left(\dfrac{L}{2\pi}\right)^2$

$A = \dfrac{L^2}{4\pi} = \dfrac{L^2}{12.566}$

TO MAX AREA:

Area of circle is larger than Area of square when perim/circumference is L. So only make a circle!