$\star$ HW ✓

§4.2 Optimization Practice & HW

Exercises: For each of the following function use derivative techniques to:

I) Find the critical points using derivative techniques and identify them as maxima or minima.

II) Find the absolute maximum and minimum values of each function if it exists.

III) Identify the intervals on which the function is increasing or decreasing.

IV) Find points of inflection and intervals where the functions are concave up or concave down for the

SHOW ALL WORK in HW section of your notebook.

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1. $f(x) = 4x^3 + 3x^2 - 6x + 1$ on the interval [-2, 1]. Use the first derivative test to justify extrema.
2. $f(x) = \sin x - \cos x$ on the interval $[0, \pi]$ Use the second derivative test to justify extrema.
3. $f(x) = \sqrt[5]{x^2}$ on the interval [-1, 32] Use the first derivative test to justify extrema.
4. $f(x) = x - \ln x$ on the interval [0.1, 5] Use the second derivative test to justify extrema.

DAY 62 & Textbook p. 202

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5. $f(x) = x + \dfrac{32}{x^2}$ over all real numbers Use the first derivative test to justify extrema.
6. $f(x) = 2x - e^x$ on the interval [-2, 4] Use the second derivative test to justify extrema.
7. $f(x) = 3x\sqrt[3]{x} - 2x$ on the interval [0, 3] Use the first derivative test to justify extrema.
8. $f(x) = \dfrac{x^4 + 1}{x^2}$ Use the second derivative test to justify extrema.

DAY 63 & Textbook p. 202

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9. $f(x) = e^x - \ln x^2$ Use the first derivative test to justify extrema.
10. $f(x) = 3\sqrt[3]{x} - 2x$ for the interval [-2, 3] Use the second derivative test to justify extrema.

In exercises 11 and 12, the derivative of the function $y = f(x)$ is given. At what points, if any, does the graph have a relative minimum or relative maximum?

11. $\dfrac{dy}{dx} = (x-1)^2(x-2)$
12. $\dfrac{dy}{dx} = (x-1)^2(x-2)(x-4)$

DAY 64 & Textbook p. 202

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§4.2 OPTIMIZATION NOTES HW $\boxed{\text{DAY 64}}$ #9–12

(9) $f(x) = e^x - \ln x^2$

$f(x) = e^x - 2\ln x$

$f'(x) = e^x - \frac{2}{x} = 0$

$f'(x) = 0$ TI-84 solve.

$x \approx 0.8526$ $\quad e^x = \frac{2}{x}$

$f'(x)$ und $x = 0$ $\quad xe^x = 2$

f': $+$ | $\emptyset$ | $-$ | 0 | $+$

$\nearrow$ 0 $\searrow$ 0.8526 $\nearrow$

$\boxed{\text{TABLE}}$

x	0	0.8526
y	$\emptyset$	2.6646

$f(x)$ increasing on $(-\infty, 0)\ (0.8526, \infty)$ b/c $f'(x) > 0$

$f(x)$ decreasing on $(0, 0.8526)$ b/c $f'(x) < 0$

$f(0.8526) = 2.6646$ Rel min b/c $f'(x)$ changes $\ominus$ to $\oplus$.

$f''(x) = e^x + \frac{2}{x^2} = 0$

$f''(x)$ und $x = 0$

$f''(x) = 0 \quad x = 0.9012$

f'': $-$ | $\emptyset$ | $-$ | 0 | $+$

$\frown$ 0 $\frown$ 0.9012 $\smile$

$f(x)$ is concave down $(-\infty, 0)\ (0, 0.9012)$ b/c $f'' < 0$

$f(x)$ is concave up $(0.9012, \infty)$ b/c $f'' > 0$

$(0.9012, 2.6706)$ is a point of inflection b/c f'' changes sign $\ominus$ to $\oplus$.

(10) $f(x) = 3\sqrt[3]{x} - 2x \quad [-2, 3]$

$f'(x) = x^{-2/3} - 2 = \frac{1}{x^{2/3}} - 2$

$f'(x) = \frac{1 - 2x^{2/3}}{x^{2/3}}$ $= 0$ @ $x = \left(\frac{1}{2}\right)^{3/2}$

$= \text{und}$ @ $x = 0$

f': [$-$ | $\emptyset$ | $+$ | 0 | $-$]

-2 $\searrow$ 0 $\nearrow$ $\left(\frac{1}{2}\right)^{3/2}$ $\searrow$ 3

$f''(x) = -\frac{2}{3}x^{-5/3} = \frac{-2}{3x^{5/3}} < 0$

for all $x \neq 0$

$\boxed{\text{TABLE}}$

x	-2	0	$\left(\frac{1}{2}\right)^{3/2}$	3
y	0.2202	0	1.4142 $\sqrt{2}$	-1.672

$f(3)$ ABS MIN, $f(0)$ Rel MIN, $f(-2) = .2202$ Rel MAX

$f''\left(\left(\frac{1}{2}\right)^{3/2}\right) < 0 \quad \therefore f\left(\left(\frac{1}{2}\right)^{3/2}\right) = 1.414 = \sqrt{2}$

by 2nd Deriv Test is an ABS (Rel) MAX.

since f is concave down.

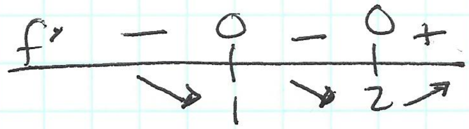
f is increasing $\left(0, \left(\frac{1}{2}\right)^{3/2}\right)$ b/c $f' > 0$

f is decreasing $(-2, 0)\ \left(\left(\frac{1}{2}\right)^{3/2}, 3\right)$ b/c $f' < 0$.

$f(x)$ is always concave down for $x \in [-2, 0)\ (0, 3]$.

b/c $f''(x) < 0$

(11) $\frac{dy}{dx} = (x-1)^2(x-2) = 0$ $\quad x=1$, $x=2$



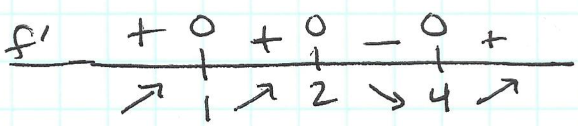
$f(x)$ increasing on $(2,\infty)$ b/c $f'>0$

$f(x)$ decreasing on $(-\infty,1)(1,2)$ b/c $f'<0$

$f(x)$ has a rel min $= f(2)$ b/c f' changes sign $\ominus$ to $\oplus$ at $x=2$.

$f(x)$ has a terrace point at $(1, f(1))$ b/c $f'<0$ on $(-\infty,1)(1,2)$.

(12) $\frac{dy}{dx} = (x-1)^2(x-2)(x-4) = 0$



$f(x)$ increasing on $(-\infty\ 1)(1,2)(4,\infty)$ b/c $f'>0$

$f(x)$ decreasing on $(2,4)$ b/c $f'<0$

$f(x)$ has terrace pt @ $(1, f(1))$ b/c $f'>0$ $(-\infty,1)(1,2)$

$f(x)$ has rel max $= f(2)$ b/c f' changes sign $\oplus$ to $\ominus$ at $x=2$

$f(x)$ has rel min $= f(4)$ b/c f' changes sign $\ominus$ to $\oplus$.