★ HW ✓

§4.2 Optimization Practice & HW

Exercises: For each of the following function use derivative techniques to:

I) Find the critical points using derivative techniques and identify them as maxima or minima.

II) Find the absolute maximum and minimum values of each function if it exists.

III) Identify the intervals on which the function is increasing or decreasing.

IV) Find points of inflection and intervals where the functions are concave up or concave down for the

SHOW ALL WORK in HW section of your notebook.

---

1. $f(x) = 4x^3 + 3x^2 - 6x + 1$ on the interval [-2, 1]. Use the first derivative test to justify extrema.
2. $f(x) = \sin x - \cos x$ on the interval $[0, \pi]$ Use the second derivative test to justify extrema.
3. $f(x) = \sqrt[5]{x^2}$ on the interval [-1, 32] Use the first derivative test to justify extrema.
4. $f(x) = x - \ln x$ on the interval [0.1, 5] Use the second derivative test to justify extrema.

DAY 62 & Textbook p. 202

---

5. $f(x) = x + \dfrac{32}{x^2}$ over all real numbers Use the first derivative test to justify extrema.
6. $f(x) = 2x - e^x$ on the interval [-2, 4] Use the second derivative test to justify extrema.
7. $f(x) = 3x\sqrt[3]{x} - 2x$ on the interval [0, 3] Use the first derivative test to justify extrema.
8. $f(x) = \dfrac{x^4 + 1}{x^2}$ Use the second derivative test to justify extrema.

DAY 63 & Textbook p. 202

---

9. $f(x) = e^x - \ln x^2$ Use the first derivative test to justify extrema.
10. $f(x) = 3\sqrt[3]{x} - 2x$ for the interval [-2, 3] Use the second derivative test to justify extrema.

In exercises 11 and 12, the derivative of the function $y = f(x)$ is given. At what points, if any, does the graph have a relative minimum or relative maximum?

11. $\dfrac{dy}{dx} = (x-1)^2(x-2)$
12. $\dfrac{dy}{dx} = (x-1)^2(x-2)(x-4)$

DAY 64 & Textbook p. 202.

---

⑤ $f(x) = x + \dfrac{32}{x^2}$ $x \in \mathbb{R}$, $x \neq 0$

$f'(x) = 1 - 64x^{-3}$
$1 - \dfrac{64}{x^3}$

$f'(x) = \dfrac{x^3 - 64}{x^3} = \dfrac{(x-4)(x^2+4x+16)}{x^3}$

C.P. $\begin{cases} f'(x) = 0 & x = 4 \\ f'(x) \text{ und} & x = 0 \end{cases}$

f': $+$ $\varnothing$ $-$ 0 $+$
↗ 0 ↘ 4 ↗

f(x) increasing on $(-\infty, 0)(4, \infty)$ b/c $f' > 0$
f(x) decreasing on $(0, 4)$ b/c $f' < 0$

$f(4) = 6$ Rel min b/c f' changes sign ⊖ to ⊕.

$f''(x) = +3(64)(x^{-4})$
$f''(x) = \dfrac{192}{x^4} > 0$ for all $x \neq 0$

f'': $+$ $\varnothing$ $+$
0

f(x) is always concave up $(-\infty, 0), (0, \infty)$ b/c $f''(x) > 0$

f(x) has no inflection points.

⑥ $f(x) = 2x - e^x$ $[-2, 4]$

$f'(x) = 2 - e^x = 0$
$e^x = 2$
$x = \ln 2$

f': $[$ $+$ 0 $-$ $]$
-2 ↗ ln2 ↘ 4

TABLE x	-2	ln2 (.693)	4
y	$-4 - \frac{1}{e^2}$ -4.135	$\ln 4 - 2$ $-.6137$	$8 - e^4$ -46.598

2nd Deriv Test
$f''(\ln 2) = -e^{\ln 2} = -2 < 0$
$\therefore$ $y = \ln 4 - 2$ ($y \approx -0.6137$) is an ABS/REL MAX b/c $f'' < 0$ at critical point.
$f(-2)$ & $f(4)$ are rel MIN & $f(4) = -46.598$ is ABS MIN.
f(x) is decreasing $(\ln 2, 4)$ b/c $f' < 0$
f(x) is increasing $(-2, \ln 2)$ b/c $f' > 0$

$f''(x) = -e^x = 0$
never
$f''(x) < 0$ always so f(x) is always concave down.
f(x) has no inflection points.

§4.2 OPTIMIZATION NOTES $\boxed{\text{DAY 63}}$ #7, 8

p15

(7) $f(x) = 3x\sqrt[3]{x} - 2x \quad [0, 3]$

$f(x) = 3x^{4/3} - 2x$

$f'(x) = 4x^{1/3} - 2$

$4\sqrt[3]{x} - 2 = 0$

$\sqrt[3]{x} = \frac{1}{2}$

$x = \frac{1}{8}$

f': $-$ 0 $+$ on $[0, \frac{1}{8}, 3]$; $\searrow$ $\nearrow$

$f''(x) = \frac{4}{3}x^{-2/3} = \frac{4}{3x^{2/3}} > 0$ for all $x \neq 0$.

$\boxed{\text{TABLE}}$

x	0	$\frac{1}{8}$	3
y	0	$-\frac{1}{16}$ -0.0625	$9\sqrt[3]{3}-6$ ≈ 6.980
		MIN ABS	MAX. ABS.

$f(x)$ increasing on $(\frac{1}{8}, 3)$ b/c $f' > 0$

$f(x)$ decreasing on $(0, \frac{1}{8})$ b/c $f' < 0$

$f(\frac{1}{8}) = -\frac{1}{16} \approx -0.0625$ is an Rel ABS MIN b/c f' changes sign $-$ to $+$.

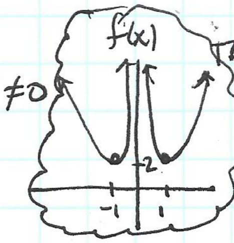
$f(3) \approx 6.980$ is ABS MAX.

$f(0) = 0$ Rel max.

$f(x)$ concave up for all $x \in [0, 3]$ b/c $f''(x) > 0$.

$f(x)$ has no points of inflection b/c f'' has no change in sign.

(8) $f(x) = \frac{x^4+1}{x^2} = x^2 + x^{-2}$ $\quad x \neq 0$



$f'(x) = 2x - \frac{2}{x^3} = \frac{2x^4 - 2}{x^3}$

$f'(x) = 0 \quad 2(x^4 - 1) = 2(x-1)(x+1)(x^2+1) = 0$ @ $x = \pm 1$

$f'(x)$ und @ $x = 0$

f': $-$ 0 $+$ $\varnothing$ $-$ 0 $+$ at -1, 0, 1; $\searrow$ $\nearrow$ $\searrow$ $\nearrow$

$f''(x) = 2 + \frac{6}{x^4} = \frac{2x^4+6}{x^4} = \frac{2(x^4+3)}{x^4}$

$f''(x) = 0$ never

$f''(x)$ und $x = 0$

f'': $+$ 0 $+$; $\cup$ $\cup$

TABLE

x	-1	0	1
y	2	und	2

2nd Deriv Test @ C.P.

$f''(-1) = 8 > 0$ $\therefore f(-1) = 2$ is a MIN ABS

$f''(+1) = 8 > 0$ $f(1) = 2$ is a min. ABS.

$f(-1) = f(1) = 2$ are both rel & abs minimums on $f(x)$.

$f(x)$ dec on $x \in (-\infty, -1)\ (0, 1)$ b/c $f' < 0$

$f(x)$ inc on $x \in (-1, 0)(1, \infty)$ b/c $f' > 0$

$f(x)$ is concave up on $(-\infty, 0)\ (0, \infty)$ b/c $f'' > 0$.

$f(x)$ has no inflection points.