ANSWERS

All

DAY 61

6) Find the relative extrema of $f(x)=\frac{x^3}{3}-x^2-3x$. Use the 2^{nd} Derivative Test. No calculator.

$f'(x)=x^2-2x-3$

$f'(x)=(x-3)(x+1)=0$

$x=3,-1$

CRITICAL POINTS $x=3,-1$

$f''(x)=2x-2$

$f''(x)=2(x-1)$

$f''(3)=2(3-1)=4>0 \therefore$ f(x) is ccup @ x=3, f(3) is a Rel. MIN.

$f''(-1)=2(-1-1)=-4<0 \therefore$ f(x) is ccdown @ x=-1, f(-1) is a Rel. MAX.

7) Find the relative extrema of $f(x)=(x^2-1)^{\frac{2}{3}}$. Use the 1^{st} Derivative Test. No Calculator.

$f'(x)=\frac{2}{3}(x^2-1)^{-\frac{1}{3}}\cdot 2x$

$f'(x)=\frac{4x}{3\left((x-1)(x+1)\right)^{\frac{1}{3}}}$

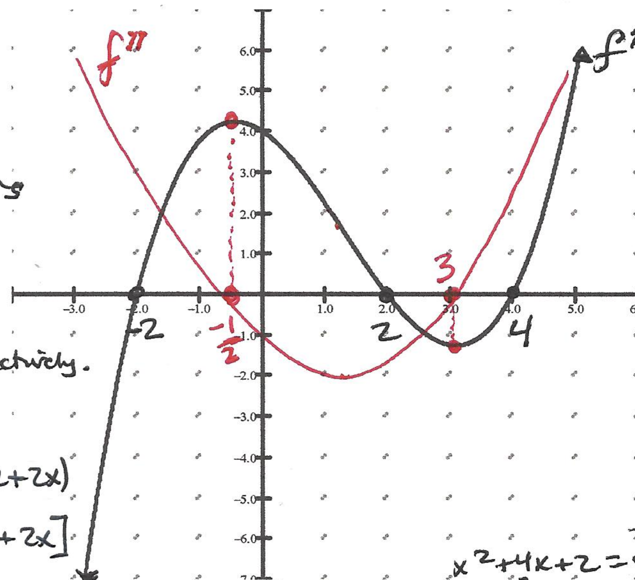
f': $-$ ∅ $+$ 0 $-$ ∅ $+$; ↘ -1 ↗ 0 ↘ 1 ↗

f' changes sign ⊖ to ⊕ @ $x=-1,1$ $\therefore f(1)$ & $f(-1)$ are Rel MIN.

f' changes sign ⊕ to ⊖ @ $x=0$ $\therefore f(0)$ is a Rel max.

C.P. $f'(x)=0$ @ $x=0$ and @ $x=\pm 1$

8) The graph of f', the derivative of a function f is shown. Find where the function f is concave up, where it is concave down and where it has points of inflection.



f(x) is concave up when $f'(x)$ has a positive slope or when $f'(x)$ is increasing or when $f''(x)>0 \therefore$ on $(-\infty,-\frac{1}{2})(3,\infty)$

f(x) ccdown (f' dec, $f''<0$) on $(-\frac{1}{2},3)$

$\therefore$ Inf. Pts @ $(-\frac{1}{2},f(-\frac{1}{2}))$ & $(3,f(3))$ b/c f'' sign changes. ⊕ to ⊖ & ⊖ to ⊕ respectively.

9) Using a calculator, find the values of x at which the graph of $y=x^2e^x$ changes concavity.

$\frac{dy}{dx}=2xe^x+x^2e^x$

$\frac{dy}{dx}=e^x(2x+x^2)$

$\frac{d^2y}{dx^2}=e^x(2x+x^2)+e^x(2+2x)$

$\frac{d^2y}{dx^2}=e^x[2x+x^2+2+2x]$

$\frac{d^2y}{dx^2}=e^x[x^2+4x+2]=0$

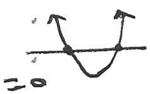
Inf Pt @ $x=-2\pm\sqrt{2}$ where f'' changes signs.

$e^x=0$ never & $x^2+4x+2=0$

$(x+2)^2-2=0$

$(x+2)^2=2$

$x=-2\pm\sqrt{2}$



10) Find the points of inflection of the following functions and determine where the function is concave up and where it is concave down. No calculator.

a) $f(x)=x^3-6x^2+12x-8$

$f'(x)=3x^2-12x+12$

$3(x^2-4x+4)$

$f''(x)=3(2x-4)$

$=6(x-2)=0$

$f''(x)=0$ @ $x=2$

f'': $-$ 0 $+$; ∩ 2 ∪

f(x) has inflection pt @ (2, 0) b/c $f''(x)$ changes sign from ⊖ to ⊕ @ x=2.

b) $f(x)=(x-1)^{\frac{2}{3}}$

$f'(x)=\frac{2}{3}(x-1)^{-\frac{1}{3}}=\frac{2}{3(x-1)^{\frac{1}{3}}}$

$f''(x)=-\frac{2}{9}(x-1)^{-\frac{4}{3}}=\frac{-2}{9(x-1)^{\frac{4}{3}}}$

denom always >0

$f''(x)=0$ never

$f''(x)$ undefined @ $x=1$

f'': $-$ ∅ $-$; +1

f(x) has no inflection pts b/c $f''(x)$ never changes signs.