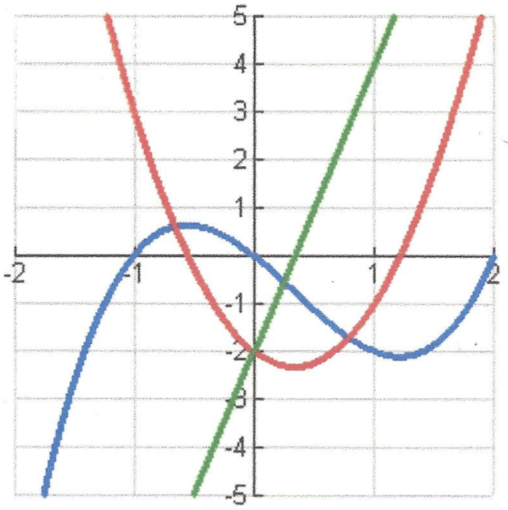
ANSWERS

§4.1 & §4.2—Day 2—Student Notes—Using the First and Second Derivatives

DAY 60



● $f(x)=x^3-x^2-2x$

● $f'(x)=$ $3x^2-2x-2$

● $f''(x)=$ $6x-2$

f'	f''
$f'>0$ 1. Positive: f is increasing	$f''>0$ 1. Positive: f' is increasing ↗ f is concave up ∪
$f'<0$ 2. Negative: f is decreasing	$f''<0$ 2. Negative: f' is decreasing ↘ f is concave down ∩
$f'=0$ 3. Intercepts: extrema on f rel max rel min	$f''=0$ 3. Intercepts: extrema on f' inflection pts on f
f' has 4. Max/Min: inflection points on f	
f' slope $=f''>0$ 5. Slope is positive: conc up on f ∪ positive on $f''>0$	
f'slope $=f''<0$ 6. Slope is negative: conc down on f ∩ negative on $f''<0$	

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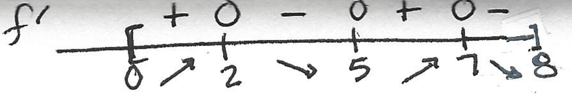
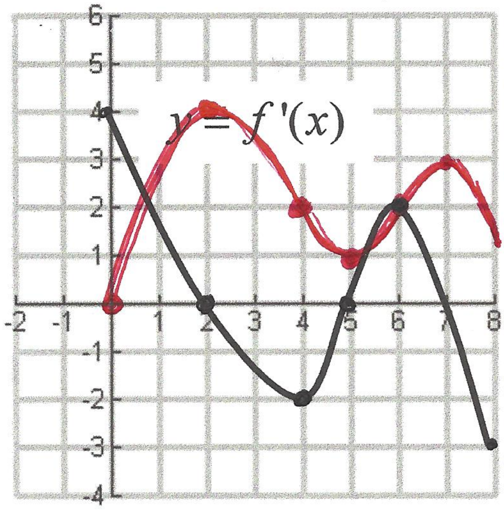


Example 1:The graph of the derivative f' of a continuous function f is shown on the interval [0,8]:



Possible f(x) graph.

a. On what intervals is f increasing or decreasing?

↗ on (0,2) (5,7) b/c f'(x) > 0   ↘ on (2,5) (7,8) b/c f'(x) < 0

b. At what values of x does f have a local maximum or minimum?

MAX @ x = 2, 7 b/c f' changes ⊕ to ⊖   MIN @ x = 5 b/c f' changes ⊖ to ⊕.

c. On what intervals is f concave upward or downward?

∪ on (4,6) b/c f'' > 0.   ∩ on (0,4), (6,8) b/c f'' < 0.

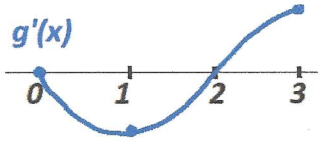
d. State the $x-$coordinates of the points of inflection.

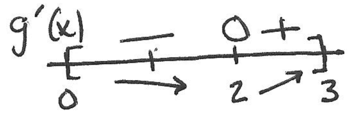
Inf. Pts @ x = 4 b/c f'' changes sign ⊖ to ⊕
@ x = 6 b/c f'' changes sign ⊕ to ⊖.

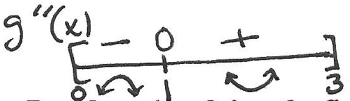
e. Assume that $\boxed{f(0) = 0}$, sketch the graph of f.

General shape inc/dec & ccu/ccd should match. y-values other than (0,0) may vary.

2. The given graph is g', state several facts about g and g''







g(x) dec on (0,2) g'<0
inc on (2,3) g'>0

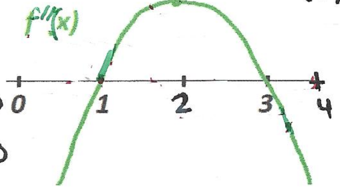
g(2) rel min b/c
g' changes ⊖ to ⊕.

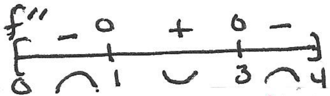
g(x) ccd on (0,1) g''<0
ccu on (1,3) g''>0

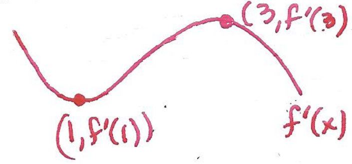
(1,g(1)) is Point of inflection b/c
g'' changes ⊖ to ⊕

3. The given graph is f'', state several facts about f and f'

(1, f(1)) is Inf Point b/c f'' changes ⊖ to ⊕
(3, f(3)) is Inf Pt. b/c f'' changes ⊕ to ⊖







f'(x) is decreasing on (−∞,1) (3,∞)

f'(x) is increasing on (1,3)

f' has a max at (1, f'(1))

f' has a min at (3, f'(3))

Problems involving the first and second derivative

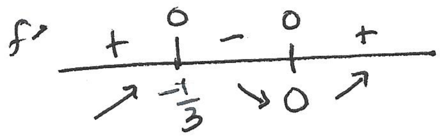
1) Find the critical numbers of each function: — f'(x) = 0 & f'(x) und

a) $f(x) = 4x^3 + 2x^2$

$f'(x) = 12x^2 + 4x$

$f'(x) = 4x(3x+1) = 0$

$f'(x) = 0 \quad \underline{x = 0} \quad \underline{x = -\frac{1}{3}}$



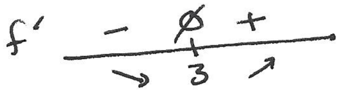
f(−1/3) is a Rel MAX b/c f' changes ⊕ to ⊖

f(0) is a Rel min b/c f' changes ⊖ to ⊕.

b) $f(x) = (x-3)^{\frac{2}{5}}$

$f'(x) = \frac{2}{5}(x-3)^{-3/5} = \frac{2}{5(x-3)^{3/5}}$

f'(x) = 0 never   f'(x) undefined $\underline{@ x = 3}$



f(3) is a Rel MIN b/c f' changes ⊖ to ⊕.

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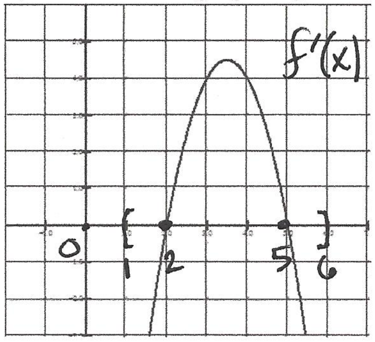
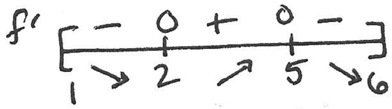
2) Shown is the graph of f' on (1,6). Find the intervals on which f is increasing or decreasing.





f is increasing on (2,5) b/c $f'(x)>0$

f is decreasing on (1,2), (5,6) b/c $f'(x)<0$.

3) Find the open intervals on which $f(x)=(x^2-9)^{\frac{2}{3}}$ is increasing or decreasing. No calculator.

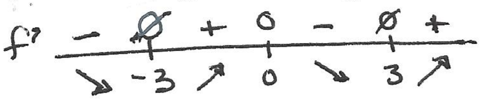
Related Poly
$+(x)(x-3)(x+3)$

$f'(x)=\frac{2}{3}(x^2-9)^{-1/3}(2x)$

$f'(x)=\frac{4x}{3[(x-3)(x+3)]^{1/3}}$

$f'(x)=0$ @ $x=0$

$f'(x)$ und @ $x=\pm 3$



f is increasing on $(-3,0)$ $(3,\infty)$ b/c $f'>0$.

f is decreasing on $(-\infty,-3)$ $(0,3)$ b/c $f'<0$.

4) The derivative of a function f is given as $f'(x)=\cos(x^2)$. Use a calculator to find the values of x on $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ such that f is increasing.

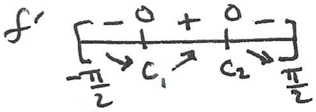
$f'(x)=\cos(x^2)\rightarrow Y_1$

on $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Find x-values for which $f'(x)>0$

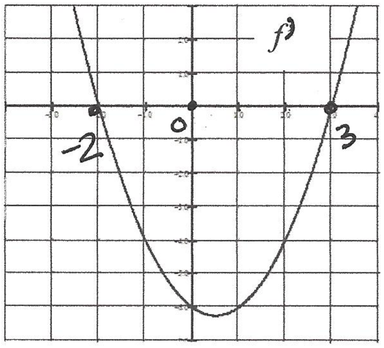
RADIAN MODE!

$f'(x)=0$ @ $x=\pm 1.2533141$



f(x) is increasing on $(-1.253, 1.253)$ b/c $f'>0$.

5) The graph of f', the derivative of a function f is shown. Find the relative extrema of f. Justify your answer.



$f'(x)=0$ @ $x=-2, 3$

$f'(x)$ changes sign ⊕ to ⊖ @ $x=-2$

∴ $f(-2)$ is a relative max.

$f'(x)$ changes sign ⊖ to ⊕ @ $x=3$

∴ $f(3)$ is a relative min.