"RECALL"

DAY 58

✱ HW ✓

R1 **HW MVT**

Write the definition of continuity.

1) $f(c)$ exists	2) $\lim_{x\to c} f(x)$ exists	3) $f(c)=\lim_{x\to c} f(x)$

R2 Write mathematical notation for differentiability: $f'(c^-)=f'(c^+)$ or $\lim_{x\to c^-} f'(x) = \lim_{x\to c^+} f'(x)=f'(c)$

R3 State the two prerequisite conditions that must be determined before the Mean Value Theorem can be applied.

1) $f(x)$ must be continuous on the closed interval $[a,b]$
2) $f(x)$ must be differentiable on the open interval (a,b)

R4 What two calculations must be determined before making a conclusion using the Mean Value Theorem.

1) $f'(c)$ exists
2) $\frac{f(b)-f(a)}{b-a}$ exists & $f'(c)=\frac{f(b)-f(a)}{b-a}$

HW. Read questions #1-4. If the function satisfies the hypotheses of the Mean Value Theorem, then solve for the value of c that satisfies the conclusion of the Mean Value Theorem. Otherwise, tell why it fails to meet the conditions of the Mean Value Theorem.

1. Given $f(x)=5-\frac{4}{x}$, find all values, c, in the interval [1,4].

$f(x)$ is continuous on $[1,4]$ & differentiable on $(1,4)$

$\therefore$ MVT applies &

$\frac{f(4)-f(1)}{4-1} = \frac{4-1}{4-1} = 1 = f'(c) = \frac{4}{c^2}$

$f'(x) = \frac{4}{x^2}$

$1=\frac{4}{c^2}$

$c=\pm 2$

$\therefore$ $\boxed{\therefore \text{By MVT } c=2,\ f'(2)=\frac{f(4)-f(1)}{4-1}=1}$

2. Given $f(x)=x^4-2x^2$, find all values, c, in the interval [-2,2].

f is continuous on $[-2,2]$ & differentiable on $(-2,2)$

$\therefore$ MVT applies &

$\frac{f(2)-f(-2)}{2+2} = \frac{8-8}{2+2} = \frac{0}{4} = 0$

$f'(x) = 4x^3-4x = 4x(x^2-1)$

$4x(x-1)(x+1)=0 \quad x=0,1,-1$

$\boxed{\therefore \text{By MVT when } c=0,-1,+1 \quad f'(c)=\frac{f(2)-f(-2)}{4}=0}$

3. Given $f(x)=x(x^2-x-2)$, find all values, c, in the interval [-1,1].

$f(x)$ is continuous on $[-1,1]$ and differentiable on $(-1,1)$

$\therefore$ MVT applies &

$\frac{f(1)-f(-1)}{1+1} = \frac{-2-0}{2} = -1$

$f'(x) = 3x^2-2x-2$

$3x^2-2x-2=-1$

$3x^2-2x-1=0$

$(3x+1)(x-1)=0$

$x=\frac{-1}{3} \quad x=1$

$\boxed{\therefore \text{By MVT when } c=\frac{-1}{3}, 1 \quad f'(c)=\frac{f(1)-f(-1)}{2}=-1}$

4. Given $f(x)=x^{\frac{2}{3}}-1$, find all values, c, in the interval [-8,8].

$f(x)$ is continuous on $[-8,8]$

but $f(x)$ is not differentiable on $(-8,8)$

b/c $f'(0)$ does not exist (sharp point on $f(x)$)

So MVT does not apply.