HW. ✯ DAY 57

		ESTIMATE TANGENT LINE APPROXIMATION ≈	$f''(a)=$	
3 $f(x)=\ln(x)$ $x=e$ $f'(x)=\frac{1}{x}$ $f'(e)=\frac{1}{e}$ $f(e)=1$	$y=\frac{1}{e}(x-e)+1$	$f(3)\approx$ $\frac{1}{e}(3-e)+1$ ≈ 1.103 ≈ 1.104	$f''(e)=$ $f'(x)=\frac{1}{x}$ $f''(x)=\frac{-1}{x^2}$ $f''(e)=\frac{-1}{e^2}<0$	f is ccd @ e $\therefore f(3)\approx 1.104$ is an overestimate.
4 $g(x)=\frac{1}{\sqrt{1+x}}$ $x=0$ $g'(x)=\frac{-1}{2(1+x)^{3/2}}$ $g'(0)=-\frac{1}{2}$ $g(0)=1$	$y=-\frac{1}{2}(x-0)+1$	$f(1.1)\approx$ $-\frac{1}{2}(.1)+1=$ $-\frac{1}{20}+1=$ $=\frac{19}{20}$	$f''(0)=$ $f'(x)=-\frac{1}{2}(x+1)^{-3/2}$ $f''(x)=-\frac{3}{4}(1+x)^{-5/2}$ $=\frac{-3}{4(1+x)^{5/2}}$ $f''(0)=\frac{-3}{4}<0$	f is ccd @ 0 $\therefore f(1.1)\approx\frac{19}{20}$ is an overestimate.
5 $h(x)=\frac{1}{1+x^2}$ $x=1$ $h'(x)=\frac{-1(2x)}{(1+x^2)^2}$ $h'(1)=\frac{-2}{4}=-\frac{1}{2}$ $h(1)=\frac{1}{2}$	$y=-\frac{1}{2}(x-1)+\frac{1}{2}$	$f(1.01)\approx$ $-\frac{1}{2}(.01)+\frac{1}{2}$ $-\frac{1}{200}+\frac{1}{2}$ $\approx\frac{99}{200}$	$f''(1)=$ $h'(x)=\frac{(-2x)}{(1+x^2)^2}$ $h''(x)=\frac{(1+x^2)^2(-2)-(-2x)(2)(1+x^2)(2x)}{(1+x^2)^4}$ $=\frac{-2(1+x^2)\left[(1+x^2)-4x^2\right]}{(1+x^2)^4}=\frac{-2(1-3x^2)}{(1+x^2)^3}$ $f''(1)=\frac{+6}{1}>0$	f is ccu at 1.01 so $f(1.01)\approx\frac{99}{200}$ is underestimate.
6 $j(x)=\cos(x)$ $x=\frac{\pi}{6}$ $j'(x)=-\sin(x)$ $j'(\frac{\pi}{6})=\frac{1}{2}$ $j(\frac{\pi}{6})=\frac{\sqrt{3}}{2}$	$y=\frac{1}{2}(x-\frac{\pi}{6})+\frac{\sqrt{3}}{2}$	$f(0.5)\approx$ $\frac{1}{2}(\frac{1}{2}-\frac{\pi}{6})+\frac{\sqrt{3}}{2}$ ≈ 0.854	$f''(\frac{\pi}{6})=$ $f'(x)=-\sin x$ $f''(x)=-\cos x$ $f''(\frac{1}{2})=-\cos(\frac{\pi}{6})$ $f''(\frac{\pi}{6})=-\frac{\sqrt{3}}{2}<0$	f is ccd at $x=\frac{\pi}{6}$ so $f(\frac{1}{2})=0.854$ is overestimate.

SAVE UNTIL DAY 59

7 SHOW WORK IN YOUR NOTEBOOK: (NEED OPTIMIZATION TO ANSWER THIS QUESTION)

Let $h(x)$ be a function defined for all $x\neq 0$ such that $h(4)=-3$ and the derivative of h is given by

$$h'(x)=\frac{x^2-2}{x} \text{ for all } x\neq 0.$$

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, local minimum or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the tangent line to the graph of h at $x=4$.

(d) Does the line tangent to the graph of h at $x=4$ lie above or below the graph of h for $x>4$? Why?